AN INVESTIGATION OF A METHOD FOR UNIQUE DETERMINATION OF AIRCRAFT LONGITUDINAL STABILITY DERIVATIVES FROM TRANSIENT FLIGHT TEST DATA

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SUMMARY

Using a North American Navion as a flight test vehicle, a study was made of the practicality of experimentally determining all longitudinal stability derivatives uniquely by releasing a large weight in flight. The solutions for the stability derivatives were attempted by matching the transient response of the aircraft with solutions to the equations of motion generated on an analog computer.

Due to instrumentation difficulties definite verification of the practicality of this procedure was not achieved. However, the experimental results indicated that the stability derivatives could be found uniquely by releasing a large weight in flight.

Unique longitudinal stability derivatives were found from simulated data generated on the analog computer. Better accuracy for these derivatives was achieved if the transient response to an elevator step was analysed in addition to the weight release response.

It is considered that this procedure for the unique determination of longitudinal stability derivatives is suitable for further investigation.

TABLE OF SYMBOLS

Capital Letters

C_L - Lift Coefficient, Lift/$_{qs}$

C_D - Drag Coefficient, Drag/$_{qs}$

C_m - Pitching Moment Coefficient, Pitching moment/$_{qsc}$

C_{L_α} - $\dfrac{\partial C_L}{\partial \alpha}$

C_{m_α} - $\dfrac{\partial C_m}{\partial \alpha}$

$C_{m_{d\alpha}}$ - $\dfrac{\partial C_m}{\partial (d\alpha)}$

$C_{m_{d\theta}}$ - $\dfrac{\partial C_m}{\partial (d\theta)}$

L - Lift, pounds

S - Wing Area, $Ft.^2$

V - True Air Speed, Ft per sec.

V_i - Indicated Air Speed, Ft per sec.

W - Aircraft Weight, Pounds

W_b - Bomb Weight, Pounds

Small Letters

c - Mean Aerodynamic Chord, Ft.

g - Acceleration of Gravity, Ft. per $sec.^2$

h - Nondimensional Longitudinal Moment of Inertia, $\dfrac{2 \rho s I_y}{m^2 c}$

Table of Symbols (Cont.)

ℓ_b	-	Distance from Aircraft cg to Bomb cg (positive with Bomb Forward), Ft.
m	-	Aircraft Mass, slugs
n	-	Perturbation Normal Acceleration (positive up), g.'s
q	-	Dynamic Pressure, $\frac{1}{2}\rho V^2$, pounds per Ft^2
d( )	-	Differential Operator, $\frac{d(\ )}{d\,{}^{t}/_{\tau}}$
$\dot{(\ )}$	-	Differentation with Respect to Real Time, $\frac{d(\ )}{dt}$

Greek Letters

α	-	Perturbation Angle of Attack, Radians
θ	-	Perturbation Angle of Pitch (positive nose up), Radians
ρ	-	Air Density, slugs per Ft^3
σ	-	Air Density Ratio, ρ/ρ_o
τ	-	Characteristic Time, ${}^{m}/_{\rho s V}$

INTRODUCTION

In recent years, dynamic flight testing has received progressively increasing emphasis. The objectives of dynamic testing are several in number. However, receiving the greatest attention is the determination of stability derivatives from the transient response of an aircraft to an applied forcing function. There are many advantages to this type of flight testing. Among the more important advantages are the short flight time and relatively simple flight techniques required. In many cases, dynamic flight testing offers the only means of finding certain stability derivatives.

The basic approach in using dynamic flight testing to determine stability derivatives is simply described. A time history of the aircraft's transient response to some input forcing function is recorded. It is assumed that the form of a set of linear, constant coefficient, differential equations describing the system is known. The time history is then used to evaluate the constant coefficients in these equations.

A basic problem that is present in dynamic flight testing is the unique determination of all longitudinal stability derivatives. The most common procedure has been to assume some theoretical relation between these derivatives. This report is concerned with the unique determination of all longitudinal stability derivatives without any such

assumptions. This is accomplished by using a weight release as a forcing function to produce the aircraft's transient response. This relieves any linear dependency between the various longitudinal stability derivatives.

This investigation was carried out at the James Forrestal Research Center, Princeton University, Princeton, New Jersey, during the 1958-1959 academic year.

EQUIPMENT

Airborne Instrumentation

The test aircraft employed was a North American Navion, which is a four place, all metal, low wing aircraft. The Navion is powered by a Continental E-185 engine rated at 185 HP at 2300 RPM and 29 inches manifold pressure. A three view drawing of the aircraft is shown in Fig. 1, and the general specifications are recorded in Table I.

The aircraft was modified by the installation of a Navy Mark 51 bomb rack which was placed on the lower center line of the fuselage, aft of the nose gear wheel well. The bomb rack was installed so that the release mechanism could be actuated either electrically by a solenoid or manually by a cable. A 566.5 pound solid steel bar was equipped to hang from the bomb rack. This steel bar was 9.5 inches in diameter and 27 inches long. Henceforth in this report, the steel bar will be referred to as the bomb. Figs. 2 and 3 show the bomb installed on the aircraft.

Standard aircraft instruments were used to determine altitude and airspeed. In addition, the aircraft was instrumented to measure normal acceleration, pitch rate and angle of attack. These variables were fed to a telemetering system and recorded at a ground station. Also, the instant of bomb release was recorded. Fig. 6 is a schematic wiring diagram for the instrumentation system.

The accelerometer used was a Giannini Model 25113-1-20 with a range of ± 1 g. The sensitive axis of this instrument was canted laterally at 45°. With this arrangement, the accelerometer will only respond to a 45° component of normal acceleration. This effectively multiplies the range of the instrument by the square root of two for normal accelerations. The full scale range is therefore ± 1.41 g's. An initial installation with a ± 3 g's Genesco accelerometer was made, but unsatisfactory data was obtained. No other small range accelerometer was available which could be employed with the telemetering equipment. A ± 1 g accelerometer canted laterally at 45° will of course respond to any lateral accelerations. However, the instrument provided reasonably good results. Full scale voltage for the accelerometer potentiometer was provided from the 28 volt battery bus. The resistance of the potentiometer is 2000 ohms. A bias battery was included to adjust the steady state voltage to the desired level.

The pitch rate was measured with a Honeywell gyro Air Force Part No. JG 7005 A-24. This type instrument operates with a rotor power supply of single phase, 400 cycle, 115 volt alternating current. The transducer is of the potentiometer type with a resistance of 530 ohms. The axis of the gyro was carefully checked to align with the aircraft axis. The potentiometer was supplied with five volts from the battery bus. These same five volts were supplied to the telemeter equipment as a full scale reference.

The angle of attack vane was mounted on a four foot boom which was located on the left wing tip. A balsa vane with a direct connection to a helipotentiometer was employed to reduce the moment of inertia and friction of the system. The potentiometer was supplied with 28 volts and a bias battery.

The telemeter unit was the pulse width type and was built by the Applied Science Corporation of Princeton. This unit contains 43 channels and a normal sampling rate of 20 per second on each channel. Three channels were used for angle of attack and pitch rate while six channels were used for the normal acceleration.

The telemetering unit is designed to use five volts as a full scale reference voltage. When properly instrumented, these same five volts should be the full scale reference for each transducer. Small variation of the aircraft voltage will not effect instrument calibrations with this arrangement. However, it was desired to improve the sensitivity of the angle of attack and normal acceleration measurements; therefore, 28 volts were employed as full scale. Since the telemeter will accept only zero to five volt signals, a bias battery was employed to set the steady state potentiometer inputs within this range. This has the disadvantage that a change in the 28 volt bus will change the steady state output of the angle of attack vane and the normal accelerometer. However, the sensitivity will not be effected.

Ground Station Equipment

The ground station equipment included a telemeter receiver, a magnetic tape recorder, an analog computer and a Sanborn model recorder. These components are shown in Figs. 4 and 5.

The telemeter receiver was built by the Applied Science Corporation of Princeton. This unit is designed to receive the coded output of the telemeter transmitter, decode the signals, mix channels as desired and transmit the data to a recording device.

An Ampex Model 309C dual track tape recorder was employed to record the data. This provided a convenient means of retaining data until ready for further analysis. The fact that the data was in an electrical form was a great advantage when the use of an analog computer is contemplated. All voice transmissions were also recorded on the second tape channel.

A Goodyear Aircraft Corporation Model L 3 (GEDA) linear electronic differential analyser was employed to analyse the data. This analog computer provides twenty-four stabilized DC amplifiers with an open loop gain of 5×10^7. The computer had a guaranteed accuracy of one percent. Provisions for accurately setting the computer board potentiometers was available.

A Sanborn Model 154-100 B four channel recorder was used

for visual presentation of the data. This instrument had a high natural frequency, and the low frequency response was flat to zero.

Instrument Calibration

Three components of the instrumentation system required calibration. These were the normal accelerometer, pitch rate gyro and angle of attack vane. The aircraft airspeed indication and altimeter were not calibrated as previous investigation indicated small errors in these systems.

The accelerometer was calibrated by tilting it through $\pm 90^{o}$. The angle of tilt and the percent of full scale voltage were recorded. This checked very closely with the factory calibration which was therefore accepted as valid. Fig. 7 shows the sine of the angle of tilt versus the potentiometer output in percent full scale voltage. The slope of this curve was found to be 2.115 g's for full scale voltage. Since the instrument was canted at 45^{o}, this value was divided by the sine of 45^{o} to give 2.995 g's for full scale. As installed in the aircraft 28 volts were applied across the accelerometer potentiometer with a five volt portion of this representing the telemetering full scale. As received at the telemetering ground station, full scale was therefore: $5/28 \times 2.995 =$ .534 g's.

Part way through the flight test program, it was discovered that when releasing the bomb electrically, the bomb release solenoid reduced the battery bus voltage by .6 volts. This reduced voltage lasted as long as the spring loaded bomb release switch was held down. This

effectively reduced the accelerometer zero by 12% of telemetering full scale. This is indicated by the negative peak on the flight 4 accelerometer trace in Fig. 14.

Henceforth, the bomb was released manually. However, in the two additional drops which were accomplished the accelerometer was found to have a dead zone in the steady state position. Time would not permit additional drops. Careful recalibration of the instrument showed it was linear outside the dead zone, and the zero was displaced .6 g. The new calibration is shown on Fig. 8 as .518 g's for telemeter full scale.

The dynamic characterisitics of the accelerometer were determined as follows. The instrument was placed on a board with the sensitive axis vertical. The support was then sharply removed, and the transient response of the instrument recorded during its free fall. A drop of approximately four inches was sufficient for the instrument to reach a zero g steady state. The instrument dynamic characteristics were determined by matching an analog computer trace to the instrument response. The damping ratio of .481 and the natural frequency of 6.00 cycles per second could then be found from the computer potentiometer settings.

The pitch rate gyro was calibrated by placing it on a constant speed turntable and by recording the revolutions per minute and the potentiometer output. In order to decrease the full scale range, a portion at each end of the potentiometer was shorted with metallic paint. The final calibration curve is shown on Fig. 9. Full scale deflection represents $\pm$ 8.5° per second.

The gyro dynamic characteristics were determined by deflecting the rotor and by recording its return to the equilibrium condition. The analog computer was used to determine its damping ratio of .0597 and its natural frequency of 4.58 cycles per second.

The angle of attack vane was calibrated in two ways. On the ground, a protractor was mounted in such a way that the angular deflection of the vane could be read. The angle and potentiometer output were recorded and are shown on Fig. 10. An in-flight calibration was made by using the telemetering receiver to record the vane position. With the aircraft in steady level flight, the aircraft attitude was read with a bubble level. By flying at varying speeds, a direct plot of angle of attack versus percent telemeter full scale is then available. Fig. 10 shows both calibrations. The ground calibration indicated telemetering full scale was 9.04 degrees while the inflight calibration gave 7.95 degrees for telemetering full scale. The dynamic characteristics of the instrument were not determined due to poor data obtained from this instrument.

In addition to the instrument calibration, it was necessary to determine the aircraft weight, c.g. location, and moment of inertia. Wheel scales were employed to determine the aircraft total weight. The aircraft was then placed on two wing jacks and supported at the tail with an accurate balance scale. The c.g. location was then determined from the location of the support points. These measurements were made with the pilot in the aircraft and the wheels raised. The test aircraft was equipped with a 55 pound sliding lead weight which was

mounted in a hollow tube in the fuselage and which could be moved 17.2 feet through a crank arrangement in the cockpit. The flight tests were made with the lead weight in the aft and in a mid position. The center of gravity under these conditions is 29.2 and 32.5% mac. While measuring horizontal distances with the aircraft in the flight attitude, the bomb c.g. by 3.4 and 1.1 inches for the aft and mid c.g. positions respectively. Because the bomb c.g. is at a reasonable distance vertically from the aircraft c.g., these distances are dependent upon an accurate attitude measurement of the flight conditions at the time of the bomb drop. This was accomplished by a flight with the bomb installed using a bubble level. The aircraft was then placed in the same attitude while on jacks on the ground and the aircraft and bomb c.g. positions were determined. The moment of inertia of the aircraft was determined by oscillating the aircraft about the jack points with a spring mounted on the tail. Fig. 11 shows the configuration employed. The damping under these conditions proved to be negligible. The natural frequency was determined by timing 50 cycles with a stop watch. Knowing the natural frequency and the spring constant of the system, the moment of inertia about the jack points was determined. Calibration of the springs showed them to be linear for the load range required. The moment of inertia was then transferred to the aircraft c.g. The moments of inertia about the aircraft c.g. were found to be 3536 and 3126 slug ft.^{2} with the c.g. in the aft and mid positions respectively.

PROCEDURE

The procedures employed in this investigation were basically very simple. A steady state flight condition was established; the controls were locked; and the bomb was dropped. The initial airspeed, pressure altitude, and temperature were recorded by the pilot. The transient response of the aircraft, as sensed by the instruments, was transmitted through the telemetering units and recorded on magnetic tape.

In detail, the procedures were carried out as follows. Since it was desired to recover the bomb, a water drop was impractical. In order to drop over land with reasonable safety, only low altitude drops at less than 500 feet were considered practical. In addition, a high altitude drop would require a major excavation to recover the bomb.

Since very smooth air was known to be absolutely necessary, it was decided to make all drop flights very near sunrise. With generally good weather, the air was smooth below 500 feet at approximately every other day. This gave an average of about two satisfactory mornings a week to attempt the drop. Winds of three to five miles per hour rendered conditions unsatisfactory for the low altitude test flight.

The initial intent was to secure good bomb drop data for three c. g. positions at a single airspeed. In addition, it was intended to record the aircraft response to a small elevator step for at least one set of conditions common with a bomb drop. A total of nine bomb drops were made. Due to instrumentation and weather problems, no com-

pletely satisfactory drop was accomplished. However, two drops were made where it was felt that the unknowns could be removed with reasonable confidence. Also, one elevator step was recorded that was felt to be usable. The data for these runs are shown in Figs. 14 through 16.

The pre-flight checks required for a test flight were practically non-existent. It was found desirable to check that the aircraft and instrumentation were in good condition the afternoon prior to a test flight. The weather predictions were checked and arrangements made to gain access to all necessary equipment for the drop on the following morning.

Prior to take-off, radio contact was made with the ground station to ascertain proper functioning of the telemetering installation. After take-off, the lead weight was placed in the desired location, and the aircraft was trimmed for level flight. When proper power settings were established the ground station was advised that a run was being commenced. The elevator was locked approximately 30 seconds prior to the bomb release with a clamp placed on the control column. If airspeed and attitude remained constant for the final 30 seconds and no gusts were encountered, the bomb was released. It was found that 200 to 400 feet was the most practical altitude range for bomb release. Higher altitudes made bombing accuracy uncertain. Lower altitudes were found to lack the necessary smooth air and required excessive care in maintaining safe clearances.

Once take-off was made, the bomb was dropped regardless of any instrumentation or telemetering problems. This required that streamlining be remounted on the bomb, but it was felt that it was unwise to make a landing with the bomb installed.

DISCUSSION AND RESULTS

Theory

In studying Equations of Motion of the longitudinal transient response of an aircraft, it is conventional to assume that velocity remains constant. Also, the aerodynamic forces can be expressed by a Taylor expansion about the equilibrium condition retaining only linear terms. Under these conditions Ref. 1 develops the equations of motion as follows:

$$(1) \quad \frac{C_{L_\alpha}}{2} \alpha + d\alpha - d\Theta = f_1(t)$$

$$(2) \quad C_{m_\alpha} \alpha + C_{m_{d\alpha}} d\alpha + C_{m_{d\Theta}} d\Theta - hd^2\Theta = f_2(t)$$

In addition, normal acceleration can be expressed as:

$$(3) \quad \frac{C_{L_o}}{2} n = d\Theta - d\alpha$$

The particular forcing function in this investigation can lead to some confusion in considering initial conditions and in placing the equations in non-dimensional form. In this investigation, the aircraft transient response was induced by dropping a weight with the controls fixed. Clearly, before the weight is dropped, the lift is equal to the total weight of the aircraft and bomb. However, the mass of the aircraft during the transient response does not include the bomb weight. To avoid confusion, it is best to think of the aircraft configuration for $t < 0$ to be that of the basic aircraft (no bomb) with its c.g. determined, plus the effects of the bomb acting in relation to the basic air-

craft c.g. If the airplane plus bomb configuration is trimmed at $t < 0$, then releasing the bomb is equivalent to applying an up force at the bomb c.g. Thus, one forcing function in the lift equation is an up force equal to the bomb weight, and in the moment equation, we have a positive pitching moment equal to the bomb weight times the distance between the basic aircraft c.g. and the bomb c.g.

(4) $L_o = W + W_b$ $\qquad W_b$ = bomb weight

(5) $C_{L_o} = \dfrac{2W}{\rho S V^2} + \dfrac{2W_b}{\rho S V^2}$

(6) Let: $C_{L_o}{}^{1} = \dfrac{2W}{\rho S V^2}$

It is pointed out that the initial angle of attack is prescribed by C_{L_o} as defined above. However, the aerodynamic time (τ) and the dimensionless moment of inertia (h) are defined by using the aircraft mass during the transient response. It is impossible therefore to duplicate all conditions using any forcing function other than that described previously.

In addition, it is necessary to examine the expression for normal acceleration. This is derived from the expressions developed in Ref. 1:

(7) $F_z = nW = mV(\dot{\Theta} - \dot{\alpha})$

If this expression is changed to non-dimensional form, the following results:

(8) $\quad \tau\, n W = m V (d\Theta - d\alpha)$

(9) $\quad n \dfrac{W}{\rho S V^2} = (d\Theta - d\alpha)$

(10) $\quad n \dfrac{C_{L_o}{}^1}{2} = (d\Theta - d\alpha)$

The forcing functions in dimensional form are the bomb weight in the lift equation; the bomb weight times the distance between the aircraft and the bomb center of gravity in the moment equation. In non-dimensional form, the forcing functions become:

For $t < to \qquad f_1(t) = f_2 t = 0$

For $t > to$

(11) $\quad f_1(t) = -\dfrac{W_b}{\rho S V^2} = -\dfrac{W_b}{W} \dfrac{C_{L_o}{}^1}{2}$

(12) $\quad f_2(t) = \dfrac{2 W_b \ell_b}{\rho S V^2 C} = -\dfrac{W_b}{W} C_{L_o}{}^1 \dfrac{\ell_b}{c}$

Define the quantities:

$$\Delta C_L = \frac{W_b}{W} C_{L_o}{}^1 , \quad \Delta C_m = \frac{W_b}{W} C_{L_o}{}^1 \frac{\ell_b}{c}$$

Then: $\quad f_1(t) = -\dfrac{\Delta C_L}{2}$ and $f_2(t) = -\Delta C_m$

All time histories obtained in transient responses are in real time.

$$(13)\quad \frac{C_{L_\alpha}}{2\tau}\,\alpha + \dot{\alpha} - \dot{\Theta} = -\frac{\Delta C_L}{\tau}$$

$$(14)\quad \frac{C_{m_\alpha}}{\tau^2}\,\alpha + \frac{C_{m_{d\alpha}}}{\tau}\,\dot{\alpha} + \frac{C_{m_{d\Theta}}}{\tau}\,\dot{\Theta} - h\,\ddot{\Theta} = -\frac{\Delta C_m}{\tau^2}$$

These expressions are identical to those obtained using an elevator step input except for the definitions of the forcing functions. For an elevator step input:

$$\Delta C_m = C_{m_{\delta_e}}\,\delta_e\,, \qquad \Delta C_L = \Delta C_m\,\frac{c}{l_t}$$

It is pointed out again that all conditions cannot be matched for the two types of forcing functions. In practical terms, initial angle of attack and velocity cannot be identical for the two cases at the same density altitude.

There is an additional point of view which can be taken about the initial conditions of the bomb drop. Equations (1) and (2) are perturbation equations. By considering $f_1(t)$ and $f_2(t)$ zero before the bomb drop, this establishes α_o and $d\Theta_o$ as zero. In a linear perturbation problem where the initial and final solutions are both steady state conditions, it is arbitrary as to which is considered the undisturbed state. It is equally valid, in the aircraft transient response to a bomb drop, to consider the final steady state as the undisturbed condition with α and $d\Theta$ both zero. The physical motion of the aircraft is a steady pitch rate.

This can be more clearly demonstrated with the use of equations. The variables can be expressed as the sum of a steady state and a transient solution.

$$\alpha = \alpha_{ss} + \alpha_t \qquad d\Theta = d\Theta_{ss} + d\Theta_t$$

Under the previous arrangement when $t = 0$:

$$\alpha_o = 0 \quad \alpha_{ss} = -\alpha_t \qquad d\Theta_o = 0 \quad d\Theta_{ss} = -d\Theta_t$$

If the final steady state is defined as the undisturbed condition, the equations become:

$$\alpha_{ss} = d\Theta_{ss} = 0 \qquad \alpha = \alpha_t \qquad d\Theta = d\Theta_t$$

Under these conditions $f_1(t)$ and $f_2(t)$ are zero after the bomb drop and are the negatives of the previous forcing functions before the bomb drop. That is, the aircraft before the bomb drop has a force and moment acting until release. The initial perturbation angle of attack and pitch rate are unknown in this case. An analytic solution of the equations will show that the steady state solutions in the first case are the negatives of the initial condition in the second case.

The equations for the steady state solutions can be found from equations (1) and (2) for the original arrangement.

$$(15) \quad d\Theta_{ss} = \frac{C_{L_{\alpha/2}} f_2(t) - C_{m_\alpha} f_1(t)}{C_{m_\alpha} + C_{L_{\alpha}/2} C_{m_{d\Theta}}}$$

$$(16) \quad \alpha_{ss} = \frac{f_2(t) + C_{m_{d\Theta}} f_1(t)}{C_{m_\alpha} + C_{L_{\alpha}/2} C_{m_{d\Theta}}}$$

The initial values for $d\Theta$ and α in the second case are represented by identical expressions as above. However, the forcing functions change signs.

In the real aircraft problem, there is some disadvantage in considering the final state as the undisturbed condition. The aircraft may not reach a complete steady state in the time period where velocity is essentially unchanged. There is also a mental difficulty in thinking of a steady pitch rate as an undisturbed condition in relation to level flight.

Linear Dependence

The determination of unique values for stability derivatives from flight test data is a problem that arises in any experimental program. The stability derivatives which can be determined uniquely is controlled by the linear dependence of the equations describing the system. This subject is considered in detail in Refs. 2 and 3. For the longitudinal motions considered in this report, the problem can be resolved without great difficulty.

In equations (13) and (14), it is considered that all flight variables and the forcing functions are known and are not zero. Under these circumstances, there is no linear dependency between the two equations. All stability derivatives and the dimensionless moment of inertia (h) can theoretically be determined. The ability to determine $C_{m_{d\alpha}}$ and $C_{m_{d\Theta}}$ independently by dropping a weight is the basic reason this investigation was undertaken. The

use of the elevator as a forcing function does not accomplish this for one reason. The tail force introduced into the lift equation is so small that its effect on the aircraft motion is negligible. Equation (13) may effectively be equated to zero and a relation between the variables α, $\dot{\alpha}$ and $\dot{\theta}$ results. Therefore, the variables in equation (14) are not linearly independent, and it is impossible to find all the stability derivatives in this equation uniquely. A complete development of what can be found is contained in Ref. 2.

In using a bomb drop as a forcing function, there is an additional advantage. The exact nature of the forcing functions are known as a force and a moment. With an elevator step, this is not the case. It is pointed out that the moment produced by the bomb release does not have to be known to solve the problem if h is known.

The facts stated above can be demonstrated by the use of the equations as follows. If equation (13) is used to eliminate θ from equation (14), the following results:

$$(17)\quad \left(C_{m_\alpha} + \frac{C_{L_\alpha}}{2} \; C_{m_{d\theta}}\right)\frac{\alpha}{\tau^2} + \left(C_{m_{d\alpha}} + C_{m_{d\theta}} - h\frac{C_{L_\alpha}}{2}\right)\frac{\dot{\alpha}}{\tau}$$

$$+ \left(\frac{\Delta C_L \;\; C_{m_{d\theta}}}{\tau^2}\right) - (h)\left[\ddot{\alpha} + \frac{\Delta C_L}{\tau}\right] = \frac{-\Delta C_m}{\tau^2}$$

If ΔC_L and ΔC_m are known and are not zero, all quantities in parenthesis can be determined with a time history of α, $\dot{\alpha}$ and $\ddot{\alpha}$. If h is known, it is not necessary to know the value of ΔC_m to determine these quantities. Inspection of the quantities in parenthesis shows that all derivatives are determined uniquely. However, if $\Delta C_L = 0$, as is the case for an elevator input, the derivatives cannot be found uniquely even if h and ΔC_m are both known. It is also pointed out that ΔC_m can be identically zero in the case of the bomb drop without destroying the uniqueness of the solution.

Preliminary Investigation

The previous analysis verified that a solution was possible for all longitudinal stability derivatives by dropping a weight. However, it was still necessary to verify that the transient response of the aircraft would be of sufficient magnitude that the instrumentation available could sense the flight variables with the necessary accuracy. It was determined that about 500 pounds could be carried externally on the test aircraft. This represented approximately 20% of the basic aircraft weight. It was estimated that the bomb and aircraft centers of gravity would be approximately eight inches apart. Refs. 4 and 5 provided reasonable estimates for the longitudinal stability derivatives. With this information, it was possible to estimate the response of the aircraft when the bomb was dropped.

Analog of the Test Aircraft

It was decided that the use of an analog computer was the most convenient method to make a preliminary analysis. Equations (13) and (14) were set up on the computer and the time history of the aircraft simulated flight parameters were placed on a Sanborn recorder. In addition, equation (7) was set up to give normal acceleration. See Fig. 12 for a schematic wiring diagram of the analog computer setup. This arrangement allowed variation of the forcing functions and all stability derivatives as desired. This arrangement assumed the aircraft was perfectly described by the equations and that the instruments would perfectly record the flight variables α, $\dot{\theta}$ and n.

Solution for Stability Derivatives from Analog Data

It was found that the stability derivatives could be found with reasonably good accuracy using the very simple statistical methods discussed in Ref. 6. However, it was found that greater accuracy in much less time could be achieved by curve matching two computer solutions. One person would set in arbitrary stability derivatives and produce a time history of aircraft motions. A second person would then reproduce the curves as closely as possible and determine the stability derivatives from the potentiometer settings. Excellent results were obtained in this manner for the derivative $C_{m_{d\theta}}$. Good results were found for C_{m_α} while only fair accuracy could be expected for $C_{m_{d\alpha}}$ and C_{L_α}.

It was also discovered that the variation in angle of attack would be quite small. This is of course related to the difficulties in determining C_{m_α} and C_{L_α}. Improved results could be obtained by increasing ΔC_m, that is, moving the bomb further from the aircraft center of gravity. Unfortunately, the location of the bomb on the aircraft was quite closely fixed by the landing gear wheel wells. The remaining solution was to shift the aircraft center of gravity. This is not nearly as satisfactory since, in practice, shifting weight aft in the aircraft increases its moment of inertia and rapidly decreases C_{m_α}. Both of these effects tend to reduce the size of the aircraft's response. The analog computer demonstrated that moving the aircraft c.g. aft increased the response qualitatively about half as much as moving the bomb c.g. forward. The character of the changes were distinct however.

In addition, the use of elevator input to create a sizeable moment input was investigated. It was found in this case that the response was effected most strongly by C_{L_α} and C_{m_α}. It was known from the previous theoretical discussion that no unique solution existed for C_{m_α}, $C_{m_{d\alpha}}$ and $C_{m_{d\theta}}$. However, the computer demonstrated that if a reasonable approximation was available for rate derivatives, good results could be obtained for C_{m_α} as well as C_{L_α}.

Using both type inputs, $C_{m_{d\theta}}$, C_{m_α} and C_{L_α} could be reproduced to a close percentage of the initial value. $C_{m_{d\alpha}}$ could not be reproduced with a high percentage of accuracy, but in terms of absolute value, it was well defined. This difficulty with $C_{m_{d\alpha}}$ is somewhat inherent due to its small size compared to $C_{m_{d\theta}}$ and C_{m_α}. As Ref. 7 states, the accuracy with which a stability derivative can be found is determined by the magnitude of its influence on the motion of the aircraft. $C_{m_{d\alpha}}$ has a relatively small influence on the aircraft's motions and therefore will not be found with high accuracy.

The variables which were to be measured included normal acceleration, pitch rate and angle of attack. Any two of the three will provide a complete solution as is shown by equations (1), (2), and (3). Under the conditions set into the analog computer for the bomb drop, the time histories indicated the following change from the steady state in the variables to be measured:

Normal acceleration:	0 to .4 g
Pitch rate:	0 to 6° per second
Angle of attack:	$-.5^\circ$ to $+2^\circ$

It was felt that acceleration and pitch rate could be measured in this range with reasonable accuracy. However, it was felt that angle of attack could probably not be measured with sufficient accuracy to give any useful information.

Results of Preliminary Investigation

The following facts were indicated by the use of the analog computer in the preliminary investigation.

a) The longitudinal stability derivatives could be found uniquely by dropping a weight approximately 20% of the aircraft weight.

b) The weight should be located further from the aircraft center of gravity. A distance of 1.00 to 1.75 feet would be satisfactory for the Navion. ($\frac{\Delta b}{c}$ = .176 to .305)

c) The derivative $C_{m_{d\theta}}$ would be determined most accurately. $C_{m_{d\alpha}}$ would be found with only fair percentage accuracy but good absolute accuracy. The use of an elevator step would assist in determining C_{m_α} and C_{L_α}. However, as mentioned previously, all conditions of flight cannot be duplicated for the two forcing functions.

d) It was likely that no reliable angle of attack information would be obtained.

e) The best means of obtaining a solution was probably matching analog data with the measured response of the aircraft.

The general technique for finding a set of stability derivatives by curve matching was believed to be as follows:

a) With the best available estimates, create time histories with the analog computer of all flight variables which are available from the test flight. The step forcing functions are easily created by using the reference voltage of the computer.

b) Vary each stability derivative separately and make an overlay for each derivative showing its effect on the time histories.

c) Carry out this procedure for each type of forcing function considered.

d) Use the bomb drop time history and create the best match possible.

e) Shift to an elevator step. Leave the derivatives $C_{m_{d\theta}}$ and $C_{m_{d\alpha}}$ fixed, and create the best match possible.

f) Repeat steps (d) and (e) until the best possible match is achieved.

Some small allowance could be made for the difference in flight conditions between the two forcing functions. If identical indicated air speeds are used, the initial angle of attack for the elevator step will be approximately two degrees less than that for the bomb drop. This condition is largely relieved by the fact that the angle of attack increases three to four degrees during the elevator step while it shows very small change during the bomb drop.

Results Using Flight Test Data

As previously mentioned in the report, no completely satisfactory data was obtained. Two bomb drops and one elevator step were selected as providing usable information. Figs. 14 through 16 show this data. The flight conditions for these runs are recorded in Table II.

When the bomb rack installation was completed, it was found that the aircraft and bomb centers of gravity were considerably closer together than had been initially estimated. This had a twofold disadvantage. As previously discussed, it reduces the size of the aircraft response, making the instrumentation requirements more severe. Also it makes the determination of the size of the moment forcing function more difficult. The horizontal distance between the aircraft and bomb centers of gravity depend quite strongly on the aircraft attitude when the centers of gravity are very close together horizontally in level flight. A one degree error in aircraft attitude created approximately a 20% error in the size of the pitching moment when the aircraft c.g. was completely aft. The situation was even worse as the aircraft center of gravity moved forward. Since no instrument recorded flight attitude, it was felt that no accurate moment forcing function could be determined. Measurement of aircraft attitude with the bomb installed showed a variation too large to be ignored. It was therefore decided to regard the moment input as an unknown. As brought out in the theory, the problem still has a unique solution; however practical application showed it was much more difficult.

The streamlining on the bomb was initially installed to reduce the aerodynamic drag. This was thought to be desirable in order to estimate the pitching moment more accurately. An estimate of .5 for the drag coefficient of the streamlined bomb was made based on its

cross sectional area. This estimate was taken from Ref. 1. The moment resulting from the bomb drag was determined to be quite small and therefore unimportant since the pitching moment was not accurately known anyway. The streamlining was retained throughout this investigation in an effort to reproduce results. It is not considered to be necessary for any future investigation of this nature using low speed aircraft. It should be pointed out that it has been assumed that there is no aerodynamic effect on the aircraft due to the bomb's presence.

The matching of the time histories for two bomb drops and one elevator step are shown in Figs. 17 through 19. The stability derivatives obtained through these curves are recorded in Table III.

In the analog computer solution, the dynamic characteristics of the accelerometer and pitch rate gyro were included. The computer circuit diagrams for these instruments are shown in Fig. 13. The use of filters to remove noise from the test data was considered. In general, it was found that the stray noise could be removed by eye better than by filtering when curve matching was employed as a method of solution.

The effect of each stability derivative on the computer time histories is shown in Figs. 20 through 24. It can be seen that ΔC_m, C_{L_α} and C_{m_α} are not clearly defined by the use of the bomb drop.

However, $C_{m_{d\theta}}$ and $C_{m_{d\alpha}}$ appear to be reasonably well defined. The elevator step improves the C_{L_α} and C_{m_α}, but the failure to reproduce flight conditions prevents high accuracy. Figs. 25 through 29 show the effect of each stability derivative on the response to an elevator step.

In spite of the instrumentation difficulties previously discussed, the numerical results of this investigation are considered to be reasonably good. In the elevator step shown in Fig. 19, there are no ambiguities in angle of attack or pitch rate. Table IV records the derivatives as established in this investigation and those arrived at in Refs. 4 and 5. The derivatives $C_{m_{d\theta}}$, $C_{m_{d\alpha}}$ and C_{L_α} established in this investigation fall within the band of previous evaluations. Conditions were not reproduced sufficiently to allow a good comparison for C_{m_α}. The indications are that C_{m_α} as established in this report is at least 10% high. C_{m_α} was determined with the aircraft c.g. located at 29.2% and 32.5% mac. This gives a theoretical decrease of .17 in the magnitude of C_{m_α}. The experimental decrease was .22.

The calculated and experimental values of ΔC_m for the bomb drops agreed within 15%. In both cases, the calculated values were greater. See Table III.

Proposed Improvements

The basic improvements which should be made have already been discussed. It is an obvious fact that the instruments employed should be reliable and cover the desired range without modification. If telemetering is employed, it is a great advantage to use the telemetering full scale voltage as the reference voltage for all potentiometer transducers.

It would be of considerable assistance if aircraft attitude and elevator position were also measured. While neither is required, they both help to determine the quality of the steady state established prior to the bomb release. In addition, aircraft attitude could be used to establish the moment forcing function. Elevator position would verify that the controls remained fixed during the bomb release and could be used as a forcing function if an elevator input was employed. This would also allow the determination of C_{m_δ} rather than ΔC_m for the elevator input.

The last but most important improvement has already been discussed in some detail; the bomb should be moved further from the aircraft center of gravity. The importance of this change is shown in Figs. 30 through 33 which present analog computer time histories of pitch rate and normal acceleration. The moment input here corresponds to a distance of 1.0 feet between the aircraft and bomb centers of gravity.

The effect of each stability derivative on this type of response is shown. In these comparisons, the same numerical values were used as those employed with the flight test comparisons shown in Figs. 20 through 24. With larger distance from the bomb to the aircraft c.g., the moment input could be determined accurately. These curves show that each stability derivative has a larger effect on the transient response when a larger moment input is used. The relative importance of each derivative is not greatly effected; however $C_{m_{d\alpha}}$ appears to show the greatest improvement.

There is a characteristic of these curves which is of considerable assistance in determining the derivatives by curve matching. There is a short time where C_{L_α} and C_{m_α} have no effect on the transient response. This is represented on the curves by the crossing of the variables n and $d\theta$ as C_{m_α} and C_{L_α} are varied. This means physically that the aircraft is passing through zero angle of attack. Fig. 34 shows that angle of attack initially decreases and then increases through zero to a positive steady state value. In the vicinity of the minimum, $C_{m_{d\alpha}}$ has no effect on the transient response while C_{m_α} and C_{L_α} have no effect where α passes through zero. In the flight tests, the steady state angle of attack was negative and no zero value occurred.

If the forcing functions are known accurately, approximate

values for the stability derivatives will locate the time periods where α is very small and where the minimum occurs ($d\alpha = 0$). These regions may then be used to determine $C_{m_{d\theta}}$ and $C_{m_{d\alpha}}$ independent of C_{L_α} and C_{m_α}.

The use of an elevator step to determine C_{L_α} initially is advantageous. Since an angle of attack vane can be used with this input, time histories of n and α give C_{L_α} directly. For the elevator step, equations (7) and (13) show that:

$$\frac{ng}{V} = \frac{C_{L_\alpha}}{2\tau}\,\alpha$$

With C_{L_α} known, the remaining derivatives can be determined with the analog computer as follows. If a reasonably good match is not available initially, vary all stability derivatives except C_{L_α} to achieve a fair correlation throughout. Leave $C_{m_{d\alpha}}$ fixed and vary $C_{m_{d\theta}}$ and C_{m_α} to match the regions where $d\alpha$ is very small (the minimum and the steady state of α). Then vary $C_{m_{d\alpha}}$ to match the initial portions of the time histories.

When a good match is achieved, shift to the elevator step input. Leave $C_{m_{d\alpha}}$ and $C_{m_{d\theta}}$ fixed and adjust C_{m_α} and C_{L_α} to achieve the best match for all three variables (n, α and $d\theta$). Return to the bomb input and leave C_{L_α} and C_{m_α} fixed and achieve the

best match possible varying $C_{m_{d\alpha}}$ and $C_{m_{d\theta}}$. Shift between the two forcing functions using these techniques until the best over-all match is achieved.

While the bomb drop establishes all derivatives uniquely, the use of the elevator step will considerably improve the accuracy with which C_{m_α} and C_{L_α} can be determined. The accuracy of the rate derivatives is improved only slightly. Since an angle of attack vane can be used successfully with an elevator step, an extra cross check is available.

CONCLUSIONS

The proposed method of uniquely determining longitudinal stability derivatives for slow speed aircraft is believed to be feasible. Due to instrumentation difficulties, positive verification of this is lacking. However, the numerical results for the Navion contained in Table III are considered reasonably valid.

The stability derivatives were determined uniquely from simulated data generated on the analog computer. The simplest and most accurate results were obtained by a curve matching technique.

The use of an elevator step function in addition to a weight release will greatly assist in the determination of the longitudinal stability derivatives. No consideration was given to any other types of forcing functions.

The accuracy with which each stability derivative is determined is associated with its influence on the aircraft response. The derivative most accurately determined by the release of a weight is $C_{m_{d0}}$. $C_{m_{d\alpha}}$ is uniquely found with fair accuracy by a bomb release. C_{m_α} and C_{L_α} can be found with better accuracy using an elevator step and previously determined values for $C_{m_{d0}}$ and $C_{m_{d\alpha}}$.

RECOMMENDATIONS

As a result of this investigation, the following recommendations are proposed:

1) An investigation of this nature should be continued to obtain improved experimental results. The configuration and the velocity of the aircraft should be varied in any additional investigation.

2) A normal accelerometer with a full scale range of approximately one g should be installed in an aircraft for longitudinal transient flight testing.

3) The aircraft should be instrumented to measure elevator position and attitude to properly analyse the response from a bomb release.

4) In this type of investigation in a Navion aircraft, a 500 pound weight should be at least one foot horizontally from the aircraft center of gravity.

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1. Perkins, C. D. and Hage, R. E.: Airplane Performance Stability and Control. John Wiley and Sons, New York, N. Y., 1949.

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4. Cooley, S. M., Jr.; Peterson, F. S. and Irvine, J. F.: Determination of Airplane Longitudinal Stability Derivatives from Transient Response. Princeton University Aeronautical Engineering Report No. 231, 1953.

5. Schuld, E. P. and Reinhart, L. J.: Determination of Longitudinal Stability Parameters by Steady State Flight Testing and Theoretical Calculations for the Ryan Navion. Princeton University Aeronautical Engineering Report No. 232, 1953.

6. Shinbrot, M.: A Least Squares Curve Fitting Method with Application to the Calculation of Stability Coefficients from Transient Response Data. NACA TN 2341, 1951.

7. Advisory Group for Aeronautical Research and Development Flight Test Manual, Vol. II, Stability and Control, Edited by Perkins, C. D., Princeton University, 1954.

TABLE I

Physical Characteristics of the Navion

A.	**Wing**	
1.	Total Wing Area (includes flaps, ailerons and 19.8 ft^2 of fuselage)	184.2 ft^2
2.	Span	33.38 ft
3.	Mean Aerodynamic Chord	5.70 ft
4.	Angle of Incidence	
	Root	$+2^\circ$
	Tip	-1°
5.	Twist	
	Aerodynamic	$2^\circ\ 31'$
	Geometric	3°
6.	Airfoil Section	
	Root	NACA 4415R
	Tip	NACA 6410R
7.	Aspect Ratio	6.044
8.	Taper Ratio	0.5265
9.	Dihedral	7.5°
10.	Chord	
	Root	7.2 ft
	Tip	3.92 ft
B.	**Vertical Tail**	
1.	Total Area	12.93 ft^2
2.	Stabilizer Area	6.87 ft^2
3.	Rudder Area	6.05 ft^2

TABLE I (Cont.)

4.	Airfoil Section	
	Root	NACA 0013.2 Mod.
	Tip	NACA 0012-64 Mod.
5.	Angle of Stabilizer with fuselage	2^{o} Nose left

C. Horizontal Tail

1.	Total Area	43.05 ft^2
2.	Stabilizer Area	28.95 ft^2
3.	Elevator Area	14.10 ft^2
4.	Span	13.172 ft^2
5.	Mean Aerodynamic Chord	3.34 ft^2
6.	Angle of Incidence	-4^{o}
7.	Airfoil Section	NACA 0012
8.	Aspect Ratio	4.02
9.	Taper Ratio	.67

D. General

1.	Length Overall	27.25 ft
2.	Tail Length	16.88 ft
3.	Weight (full fuel)	2430 pounds
4.	Engine	Continental E-185 185 HP at 2300 RPM, 29" Hg M.P. at sea level

TABLE II

Test Flight Conditions

	Flight 4	Flight 9	Flight 9B
Date	23 April 59	2 May 59	2 May 59
Airspeed mph	94	96	94
Temp	7° C.	9° C.	9° C.
Pressure Altitude (ft)	350	150	450
c.g.	29.2% mac	32.5% mac	32.5% mac
V_i (fps)	137.8	140.5	137.8
V (fps)	136.6	139.4	137.3
σ	1.018	1.0175	1.006
W	2540	2538	2525
C_{L_o}	.748	.720	-
C_{L_o}'	.611	.589	.607
ΔC_L	.1360	.1312	
I	3132	3541	3541
h	.0787	.0887	.0887
τ	1.296	1.273	1.298

TABLE III

Stability Derivatives from Flight Tests

	Flight 4	Flight 9 and 9B
Date	23 April 59	2 May 59
c.g. location	29.2% mac	32.5% mac
C_{L_α}	4.98	4.98
C_{m_α}	-.825	-.60
$C_{m_{d\alpha}}$	-.072	-.073
$C_{m_{d\theta}}$	-.167	-.166
ΔC_m	.00244	.00463 (Flight 9, bomb drop)
Calculated ΔC_m for Bomb Drop	.0029	.0065

TABLE IV

Comparison of Stability Derivatives

Parameter	Dynamic Flight Test (Evaluation by this Report)	Theoretical (Evaluation by Ref. 5)	Static Flight Test (Evaluation by Ref. 5)	Dynamic Flight Test (Evaluation by Ref. 4)
C_{L_α}	4.98		5.73	5.03
C_{m_α}	-0.60	-0.302	-0.370	-0.57
$C_{m_{d\alpha}}$	-0.073	-0.08	-0.06	-0.09
$C_{m_{d0}}$	-0.166	-0.17	-0.12	-0.18
V	137 fps	-	-	176 fps
C_L	.720	-	.356	.461
c.g. position	32.5%	-	32.4%	30.2%

Note: C_{m_α} corrected to 32.5% mac for all evaluations.

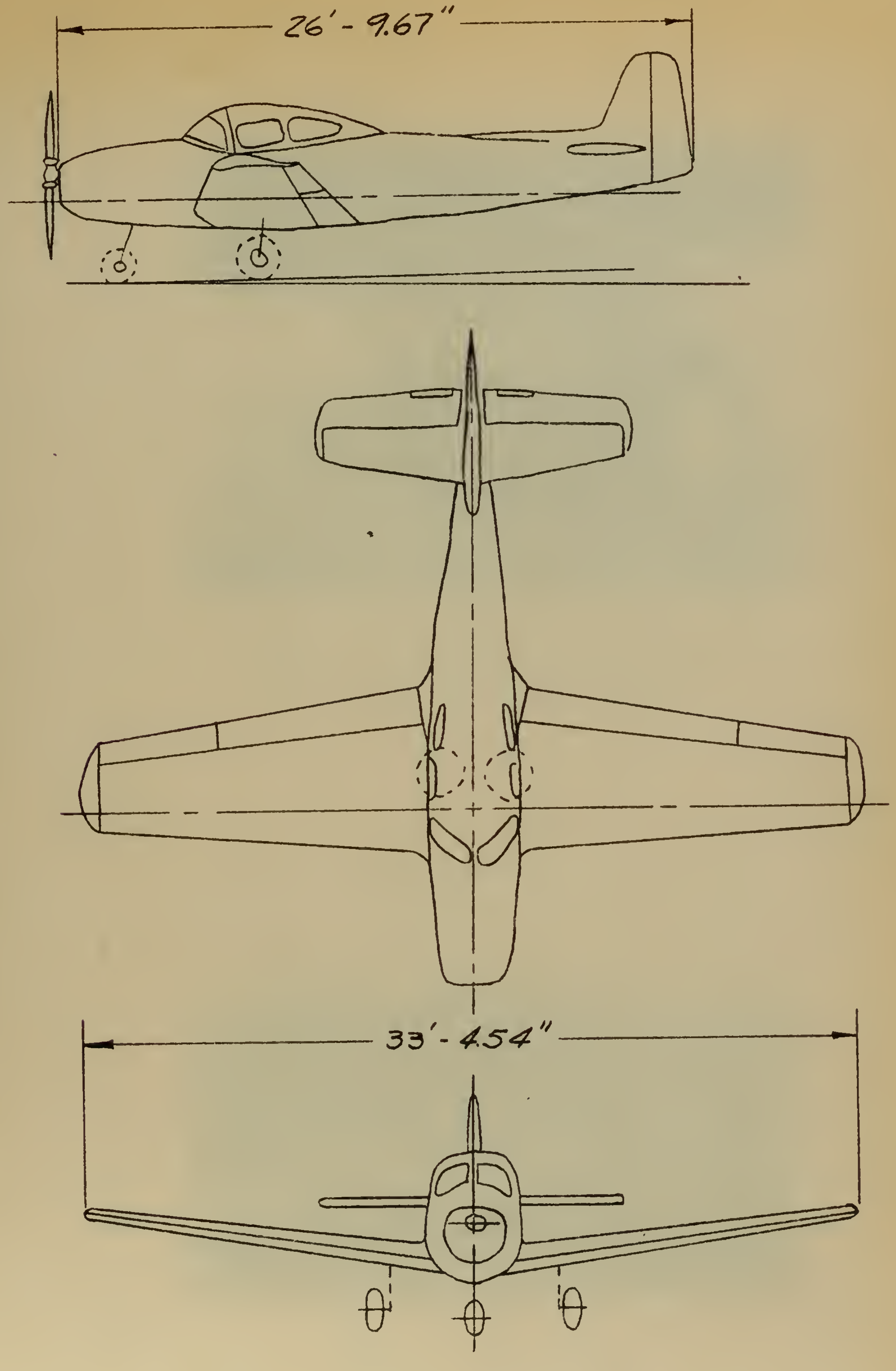


FIG. 1 THREE VIEW DRAWING

Fig. 2   Bomb Rack Installation

Fig. 3   Test Aircraft

Fig. 4 Tape Recorder and Telemeter Receiver

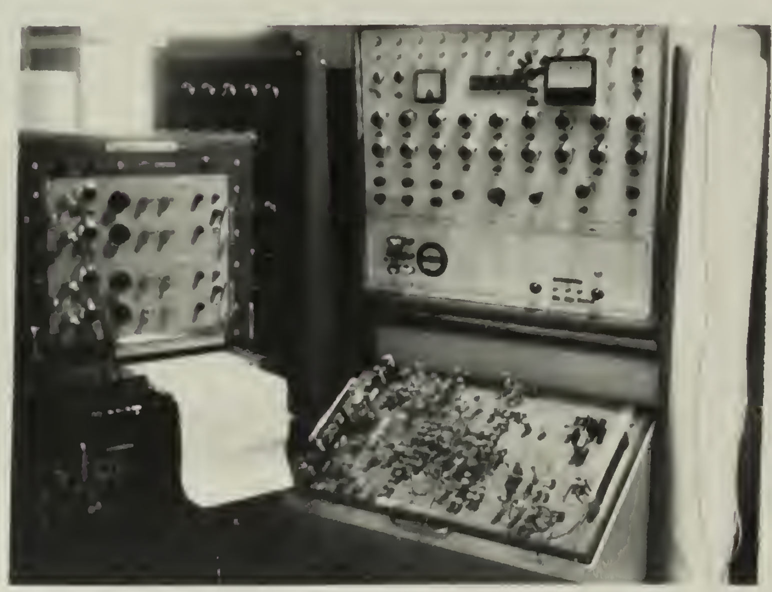


Fig. 5 Analog Computer and Sanborn Recorder

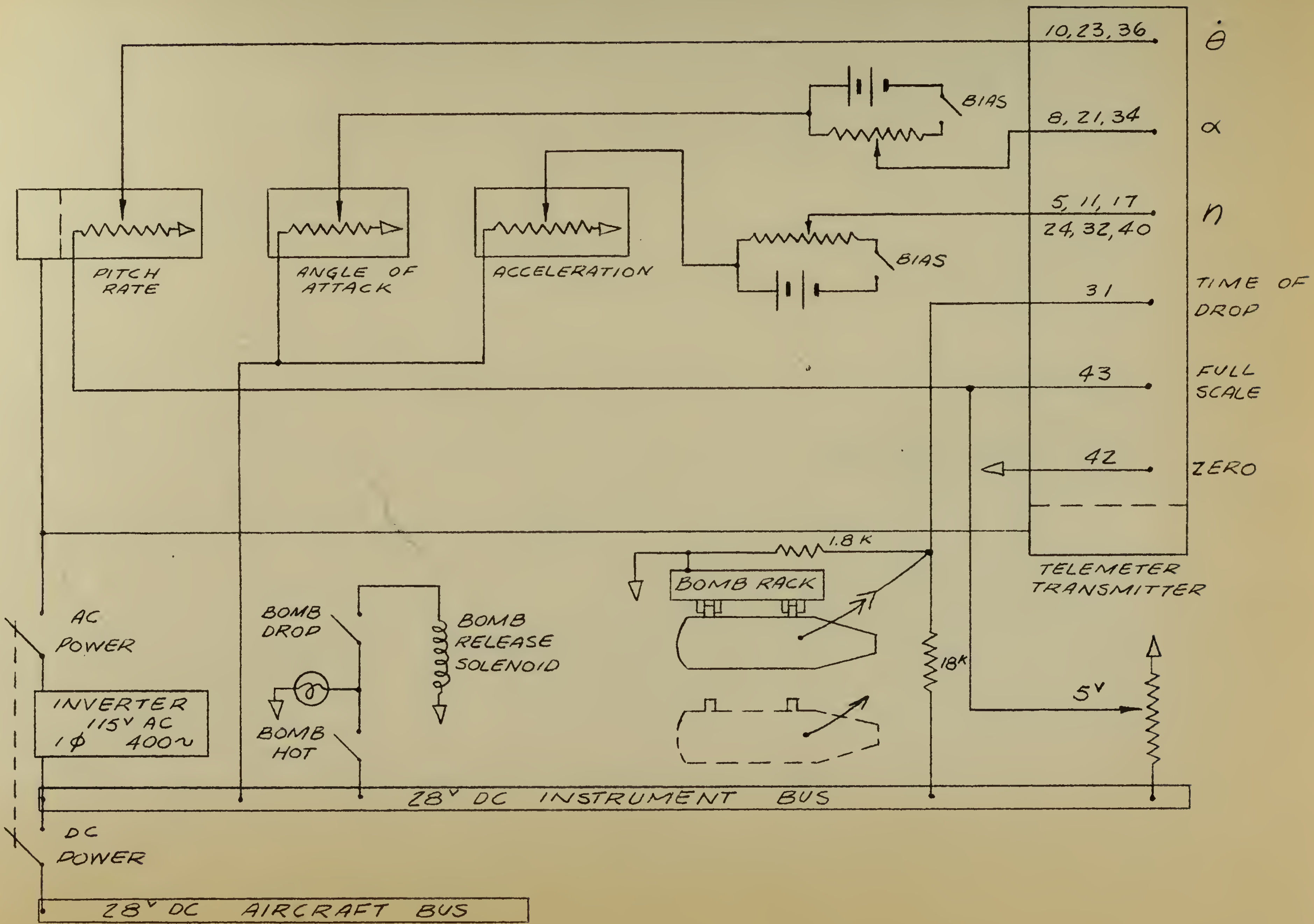


FIG. 6 AIRCRAFT INSTRUMENTATION SCHEMATIC

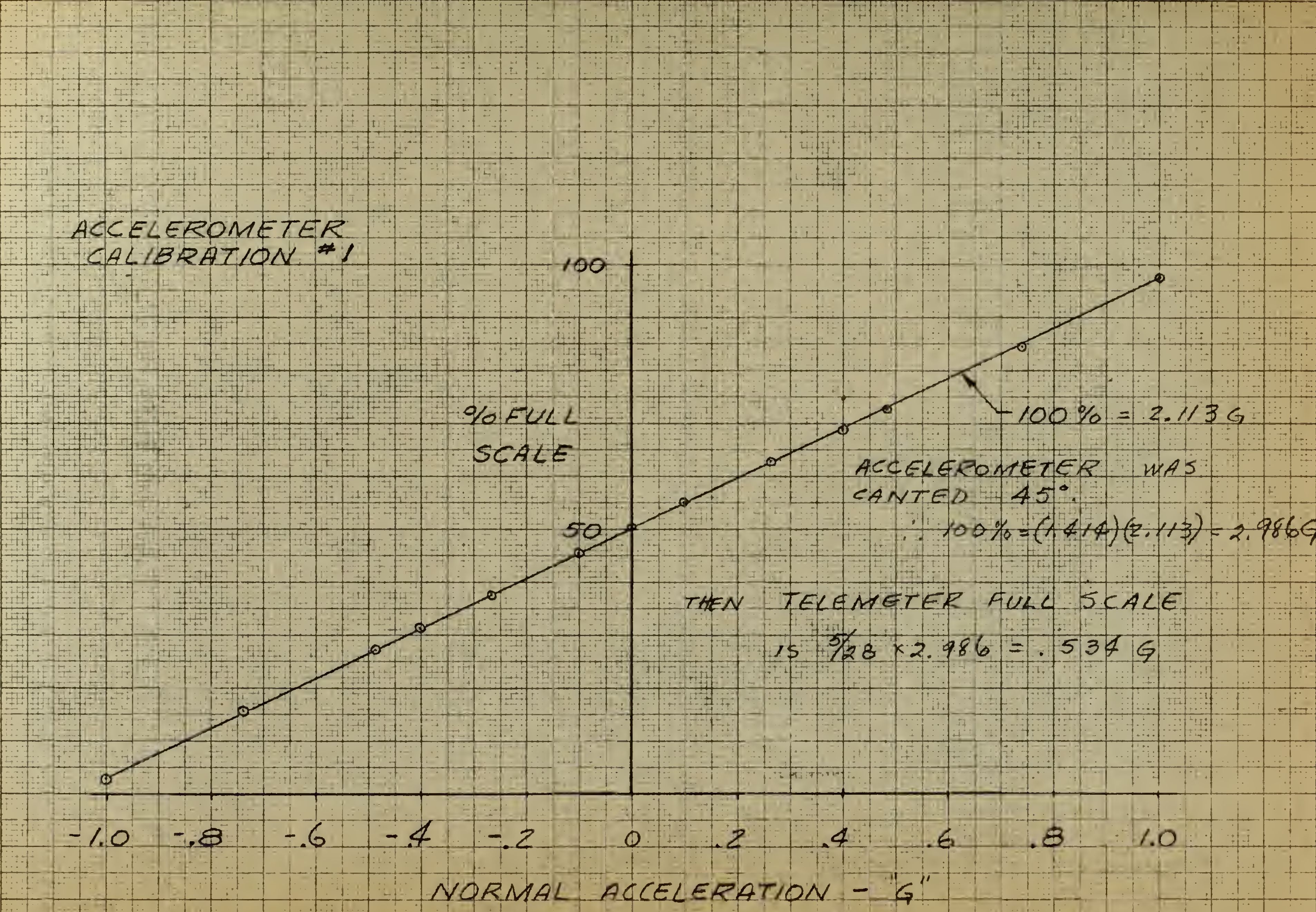

ACCELEROMETER
CALIBRATION #1
% FULL
SCALE
100
50
100% = 2.113 G
ACCELEROMETER WAS
CANTED 45°.
∴ 100% = (1.414)(2.113) = 2.986 G
THEN TELEMETER FULL SCALE
IS 5/28 × 2.986 = .534 G
-1.0
-.8
-.6
-.4
-.2
0
.2
.4
.6
.8
1.0
NORMAL ACCELERATION - "G"

FIG. 7

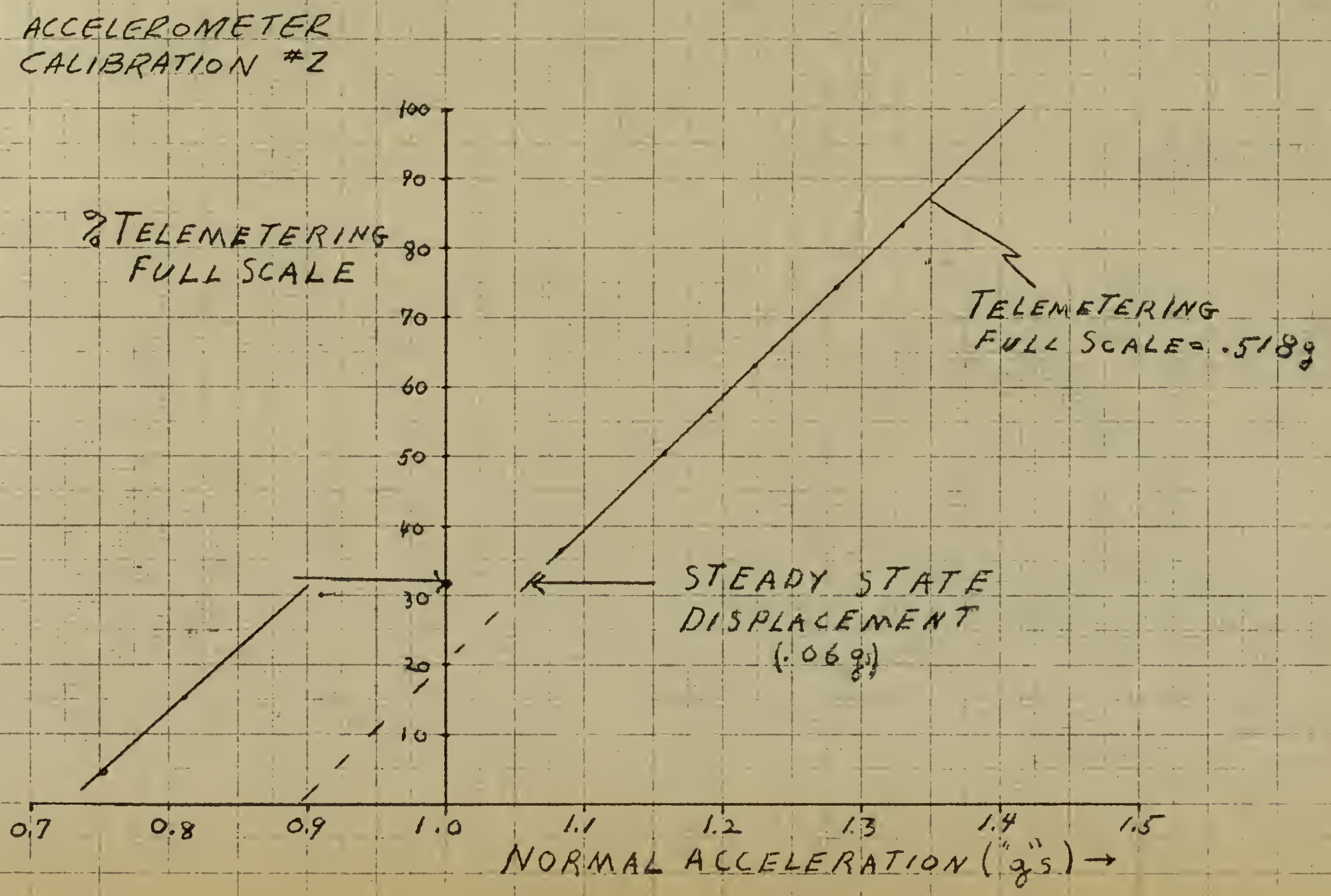

ACCELEROMETER
CALIBRATION #2
% TELEMETERING
FULL SCALE
100
90
80
70
60
50
40
30
20
10
TELEMETERING
FULL SCALE = .518 g
STEADY STATE
DISPLACEMENT
(.06 g's)
0.7
0.8
0.9
1.0
1.1
1.2
1.3
1.4
1.5
NORMAL ACCELERATION ("g"s) →

FIG. 8

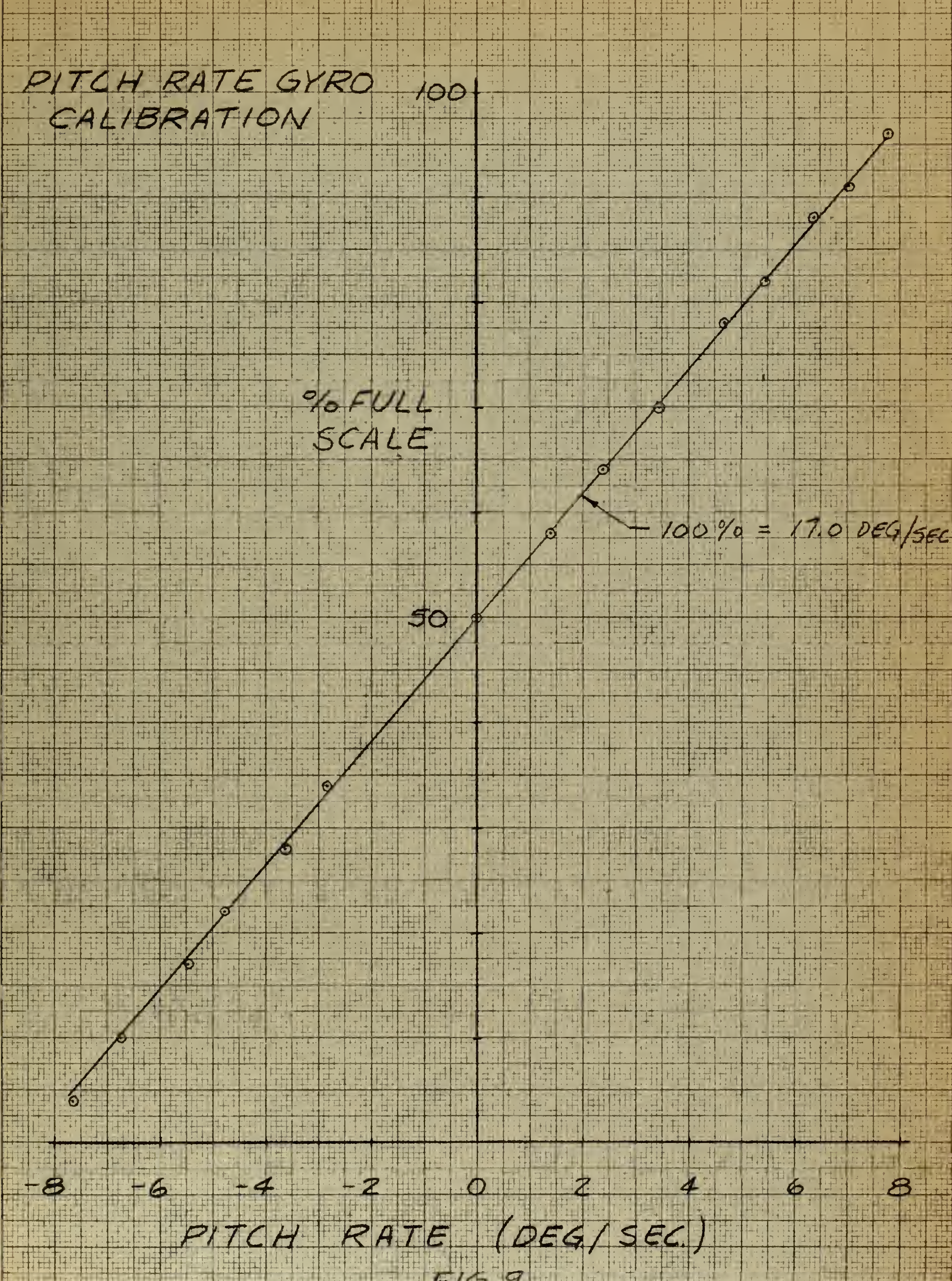

PITCH RATE GYRO
CALIBRATION
100
% FULL
SCALE
100% = 17.0 DEG/SEC
50
-8
-6
-4
-2
0
2
4
6
8
PITCH RATE (DEG/SEC.)

FIG. 9

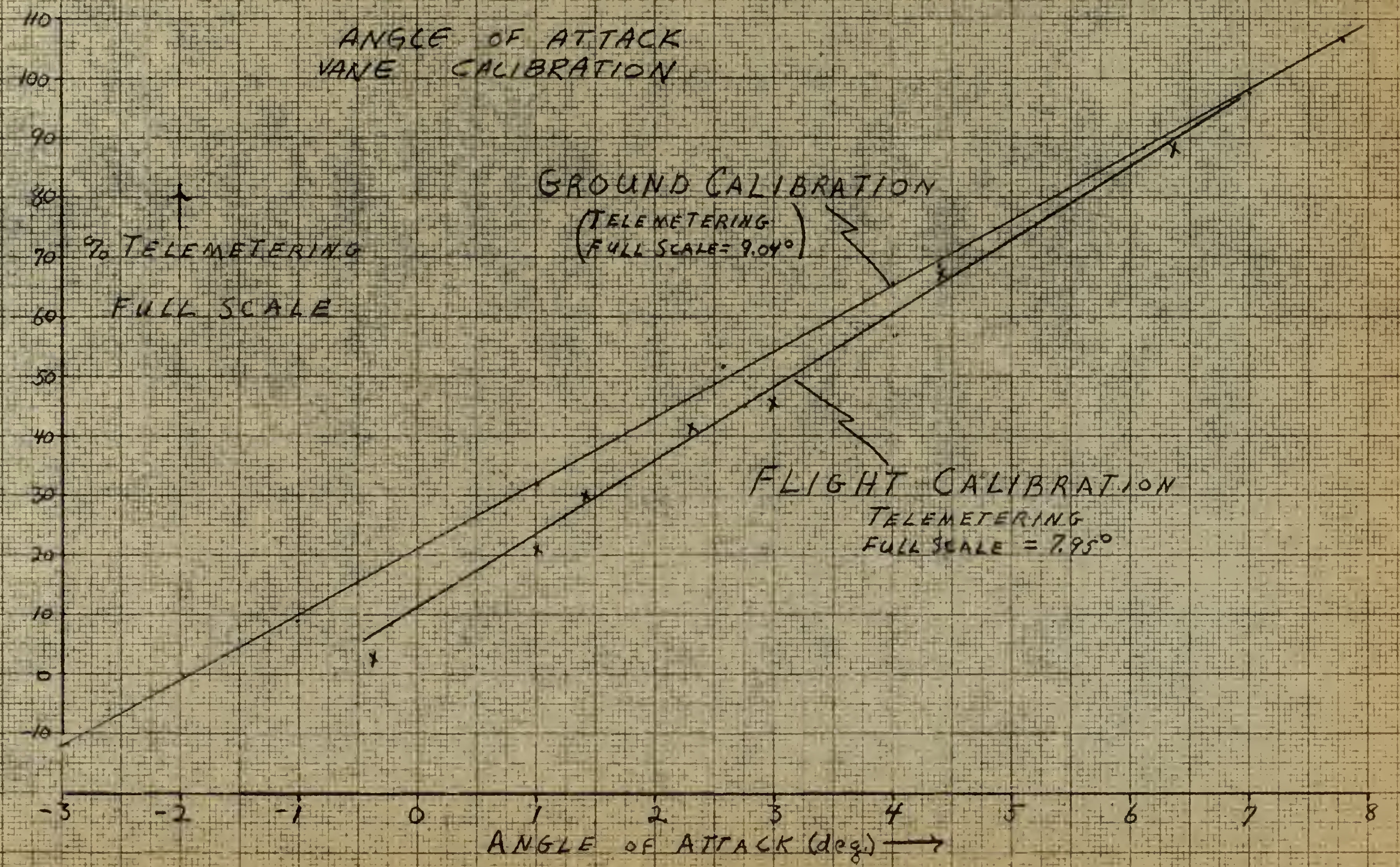
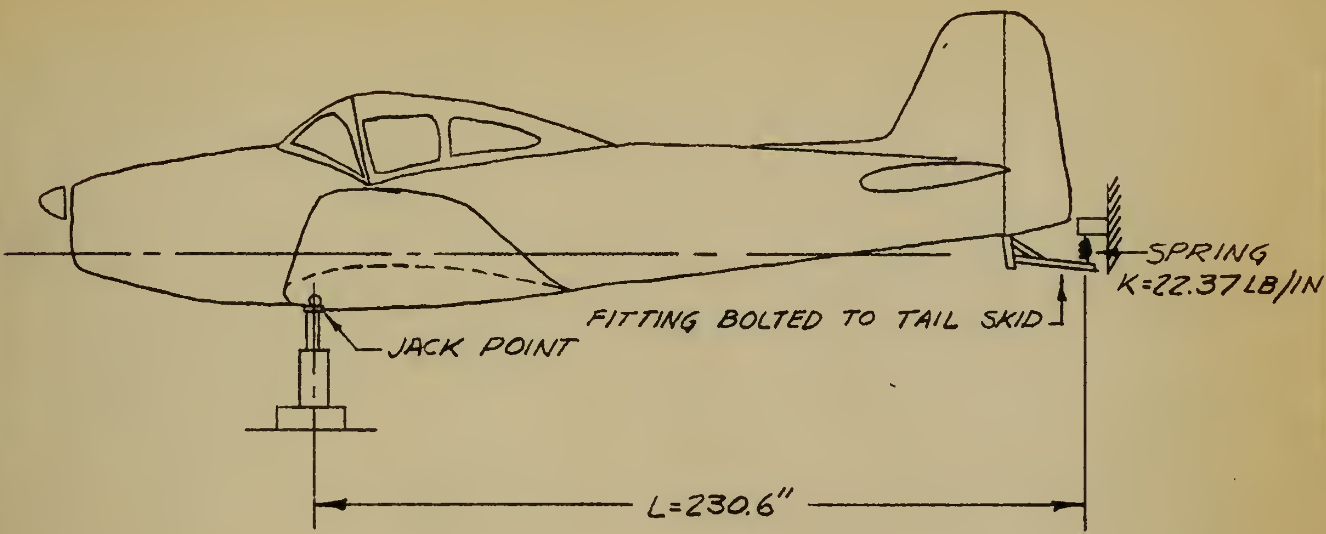

ANGLE OF ATTACK
VANE CALIBRATION
% TELEMETERING
FULL SCALE
GROUND CALIBRATION
(TELEMETERING
FULL SCALE = 9.04°)
FLIGHT CALIBRATION
TELEMETERING
FULL SCALE = 7.95°
ANGLE OF ATTACK (deg.)
110
100
90
80
70
60
50
40
30
20
10
0
-10
-3
-2
-1
0
1
2
3
4
5
6
7
8

FIG. 10



DETERMINATION OF MOMENT OF INERTIA

ASSUMING AN UNDAMPED OSCILLATION,

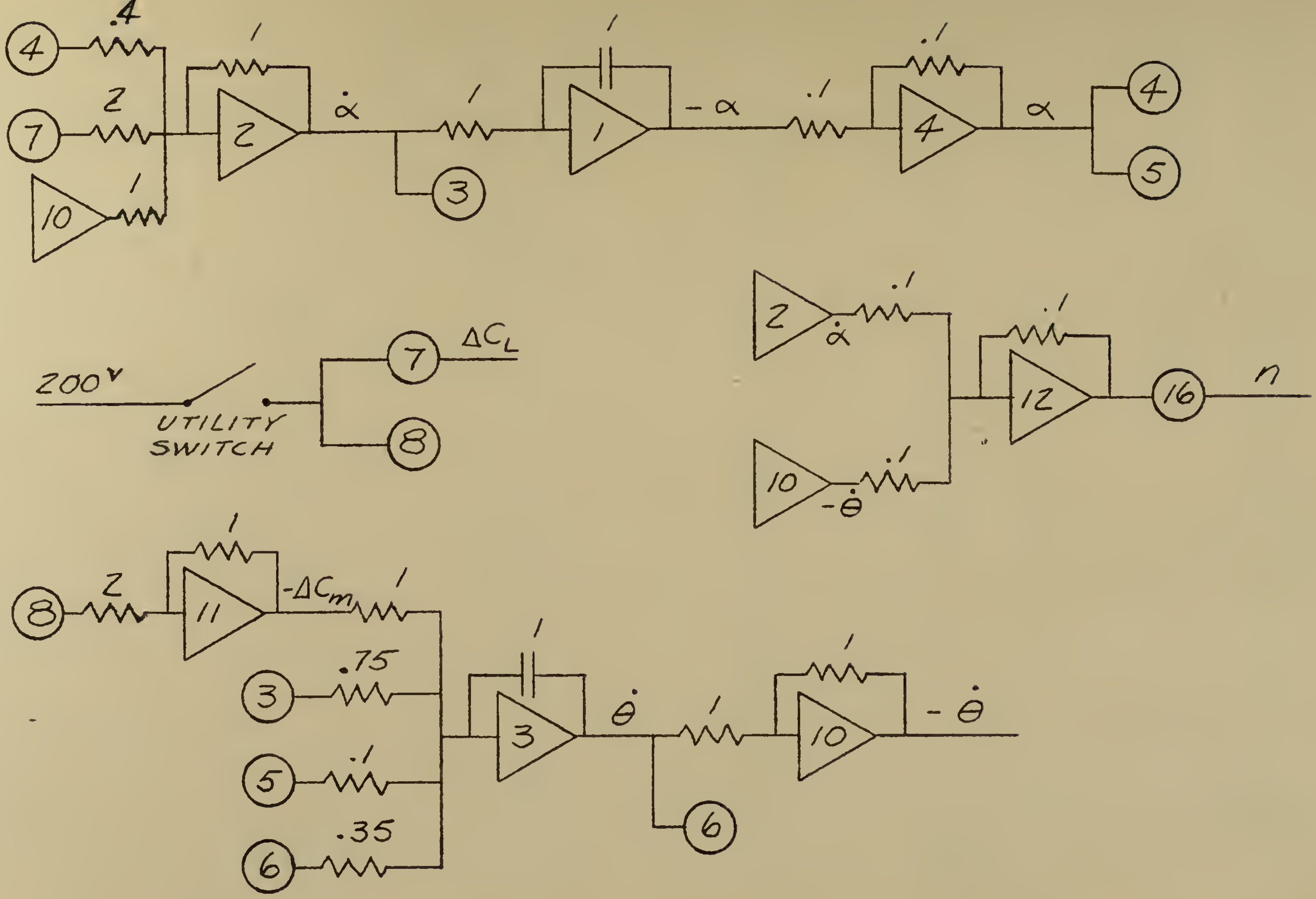
$$I = \frac{P^2 K L^2}{4\pi^2}$$

P = PERIOD (SECONDS)

FOR C.G. = 29.2 % m.a.c.
P = 1.138 SEC. I = 3240 SLUG FT² ABOUT JACK PT.

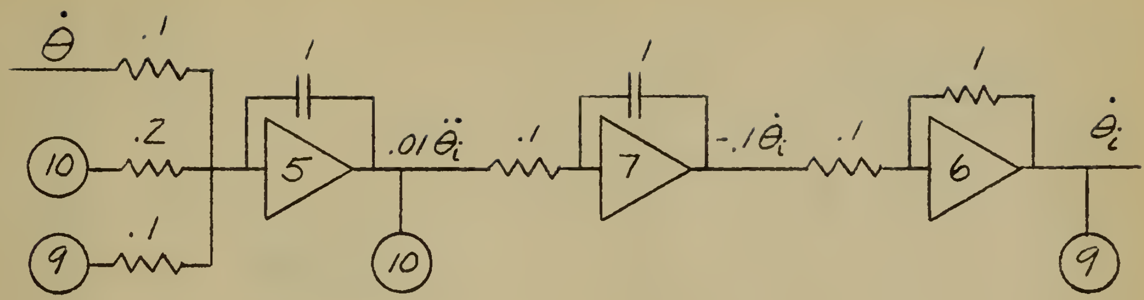
FOR C.G. = 32.5% m.a.c.
P = 1.213 SEC. I = 3690 SLUG FT² "

FIG. 11



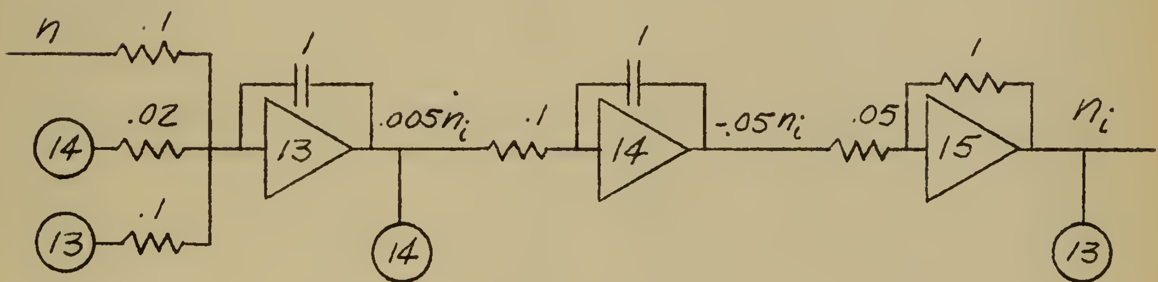
POT	3	4	5	6	7	8	16
REPRESENTATION	$\frac{.75C_{m\dot{\alpha}}}{h\tau}$	$\frac{.2C_{L\alpha}}{\tau}$	$\frac{.1C_{m\alpha}}{h\tau^2}$	$\frac{.35C_{m\dot{\theta}}}{h\tau}$	$\frac{\Delta C_L}{.4\tau}$	$\frac{\Delta C_m}{.2h\tau^2}$	$\frac{V}{10g}$

FIG. 12 ANALOG OF NAVION LONGITUDINAL SHORT PERIOD EQUATIONS OF MOTION



POT	REPRESENTATION	SETTING
9	$.01\ \omega_n^2$	.830
10	$.01\ (2\zeta\omega_n^2)$	.688

PITCH RATE GYRO INSTRUMENT



POT	REPRESENTATION	SETTING
13	$.005\ \omega_n^2$	.7105
14	$.005(2\zeta\omega_n^2)$	.725

ACCELEROMETER INSTRUMENT

FIG. 13 ANALOGS OF TEST INSTRUMENTS

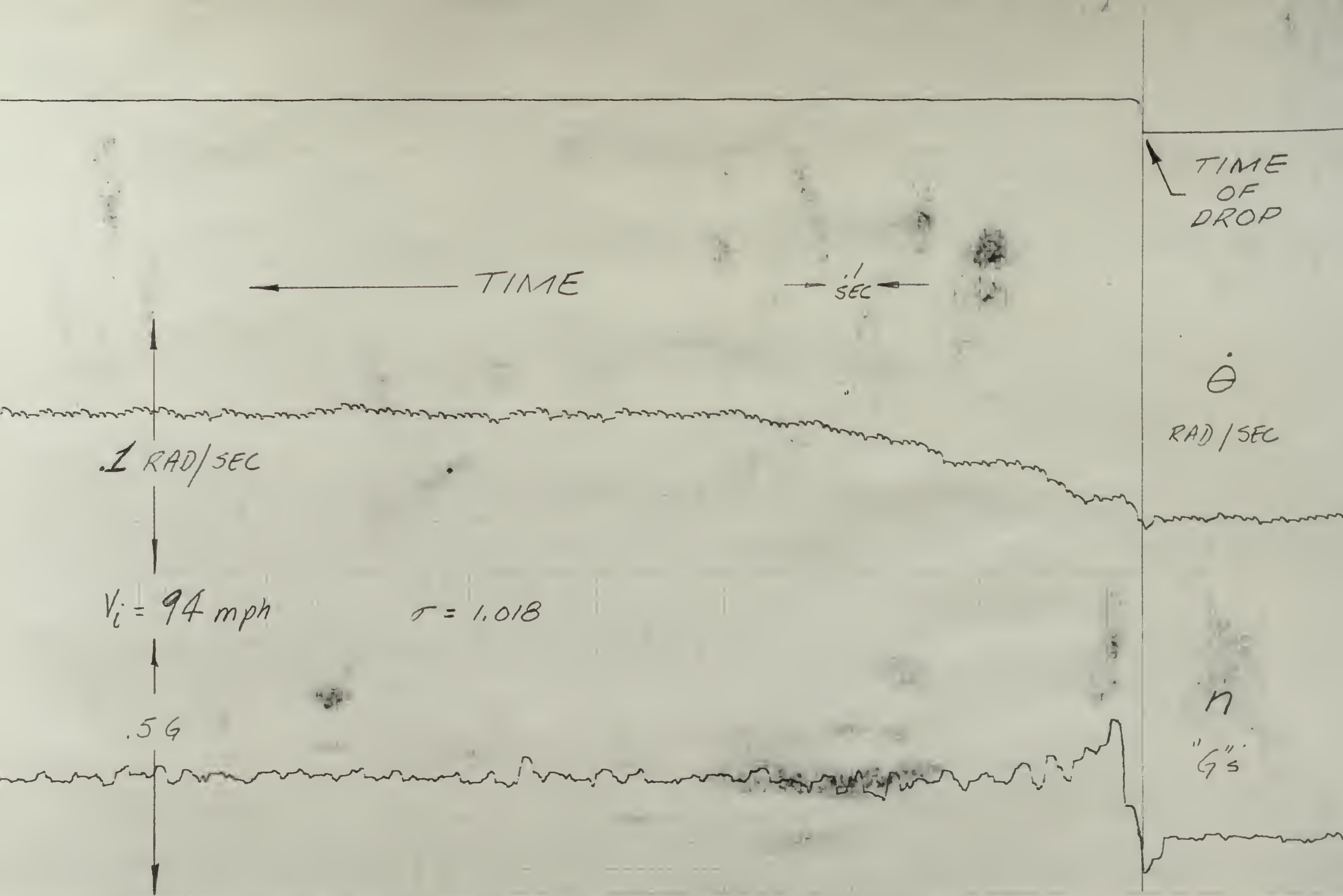


Fig. 14 Bomb Drop Flight Data Flight 4 c.g. 29.2% m.a.c.

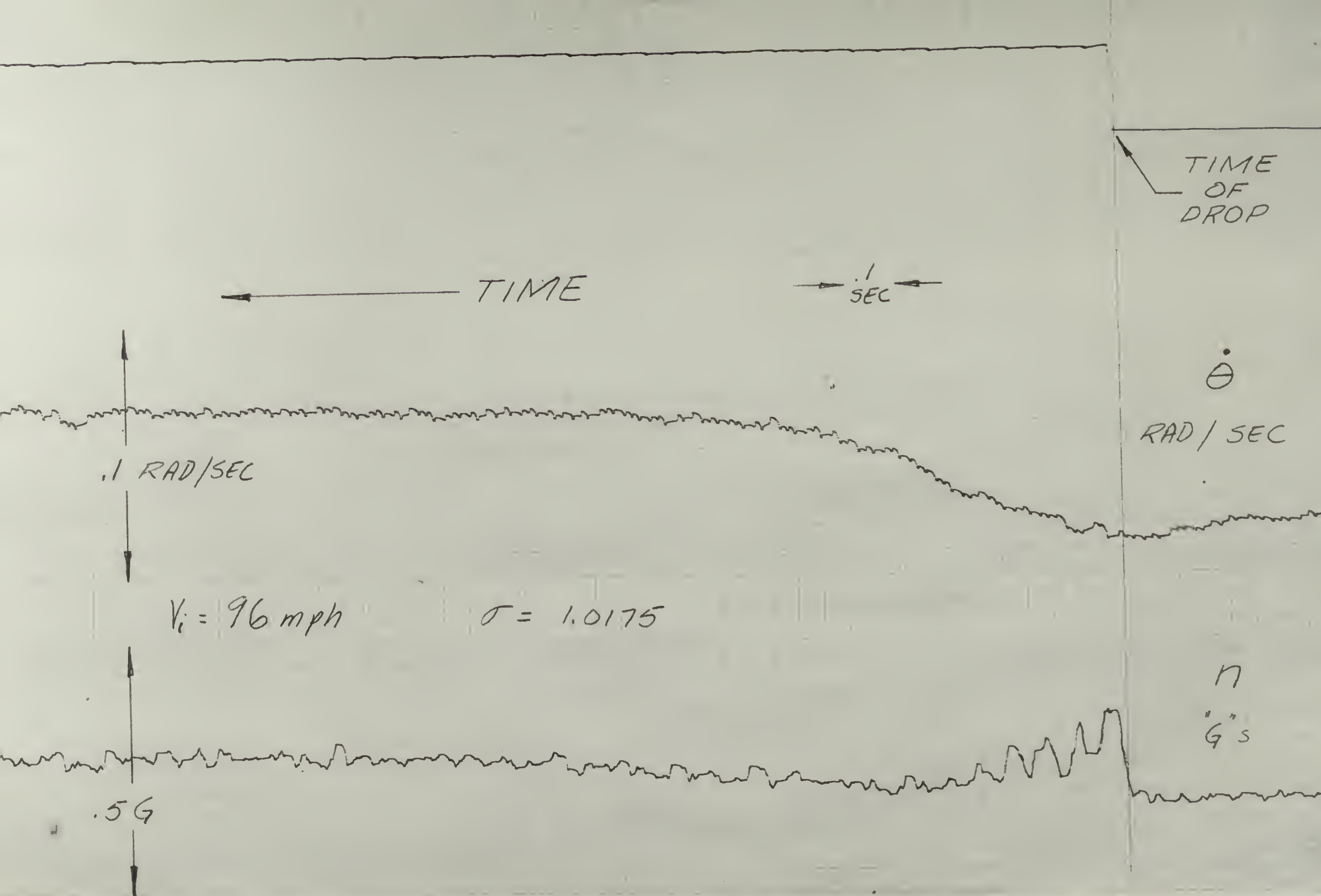


Fig. 15 Bomb Drop Flight Data Flight 9 c.g. 32.5% m.a.c.

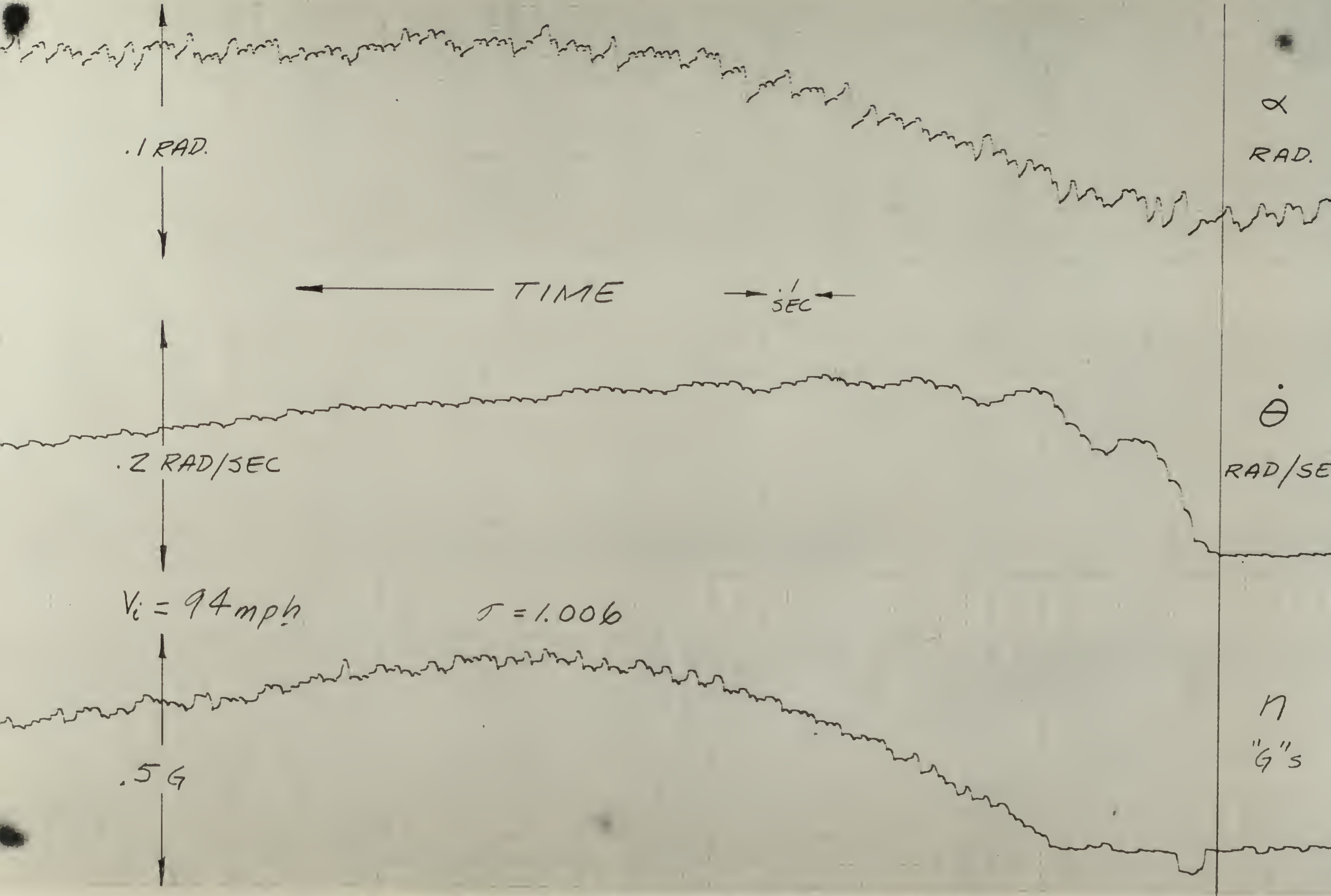


Fig. 16 Elevator Step Flight Data Flight 9B c.g. 32.5% m.a.c.

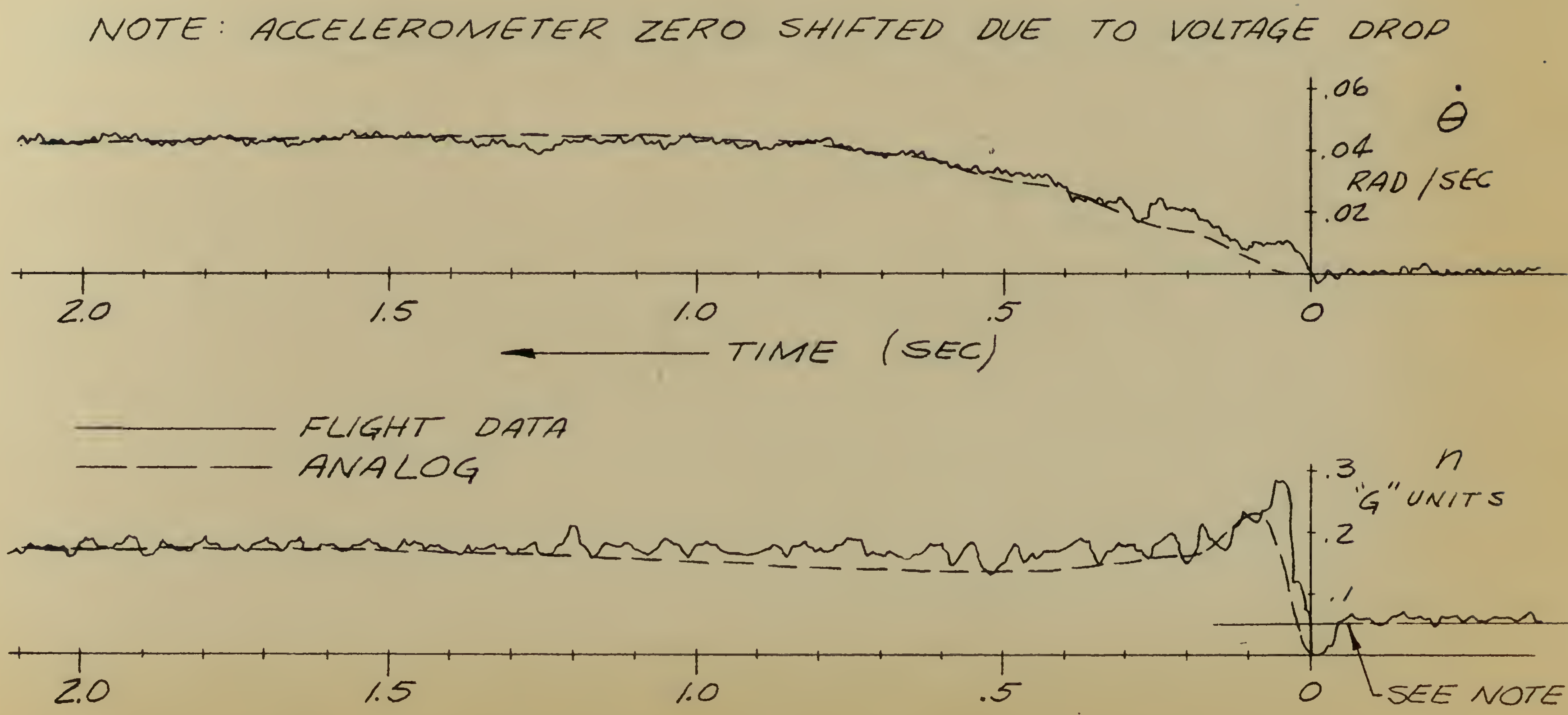


FIG. 17 RESPONSES TO BOMB DROP

FLIGHT 4 C.G. 29.2 % m.a.c.

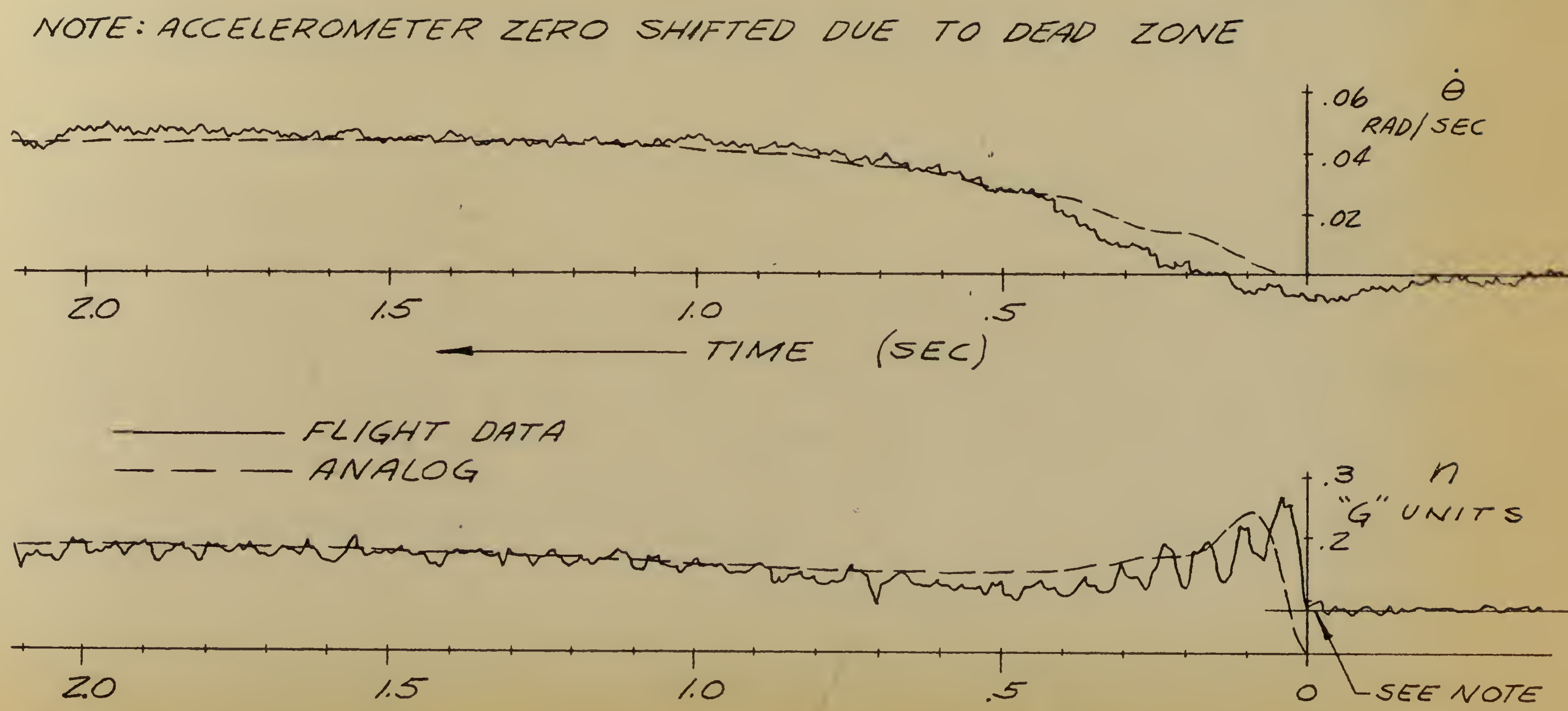


FIG. 18 RESPONSES TO BOMB DROP

FLIGHT 9 C.G. 32.5 % m.a.c.

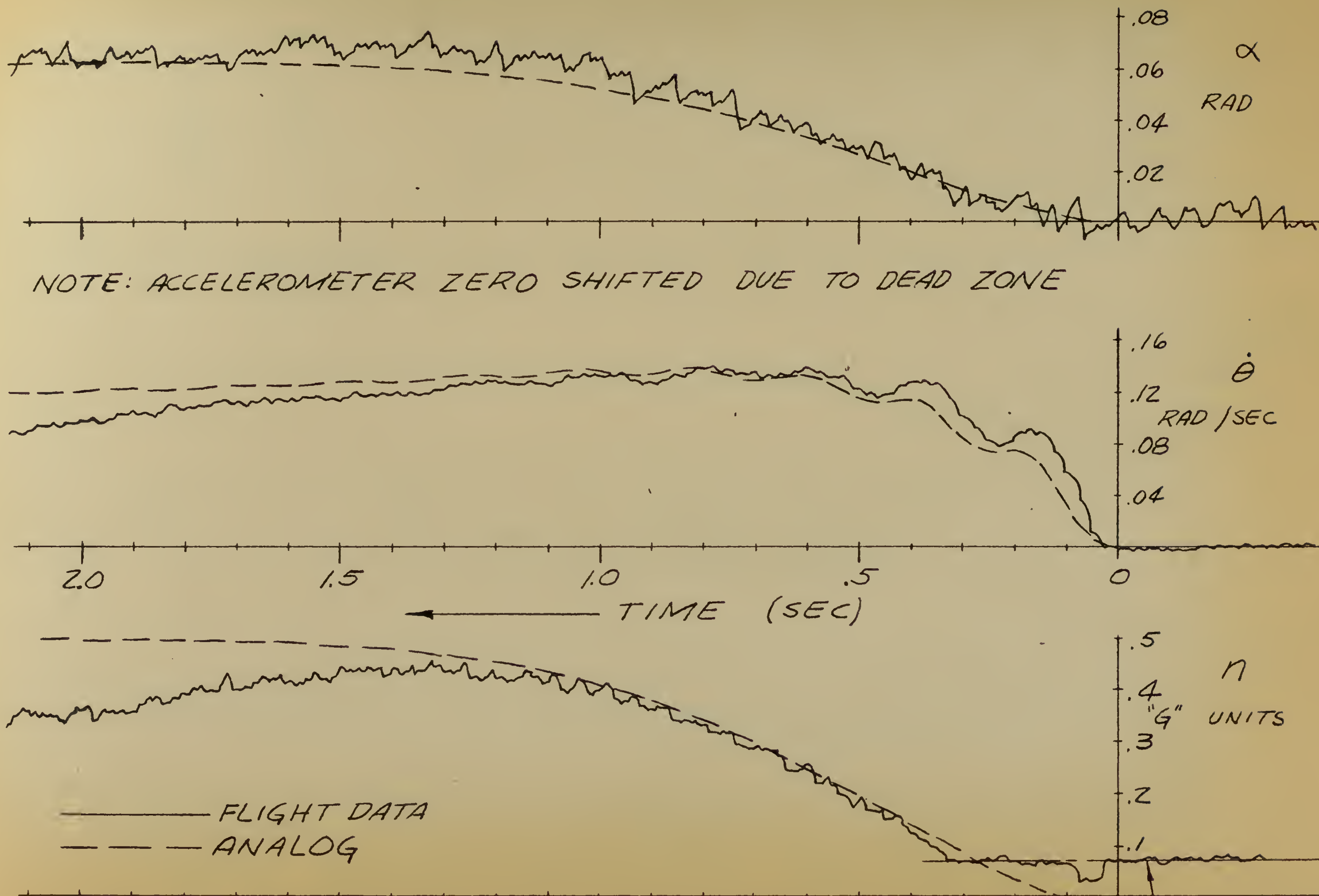


FIG. 19 RESPONSES TO ELEVATOR STEP
FLIGHT 9B C.G. 32.5% m.a.c.

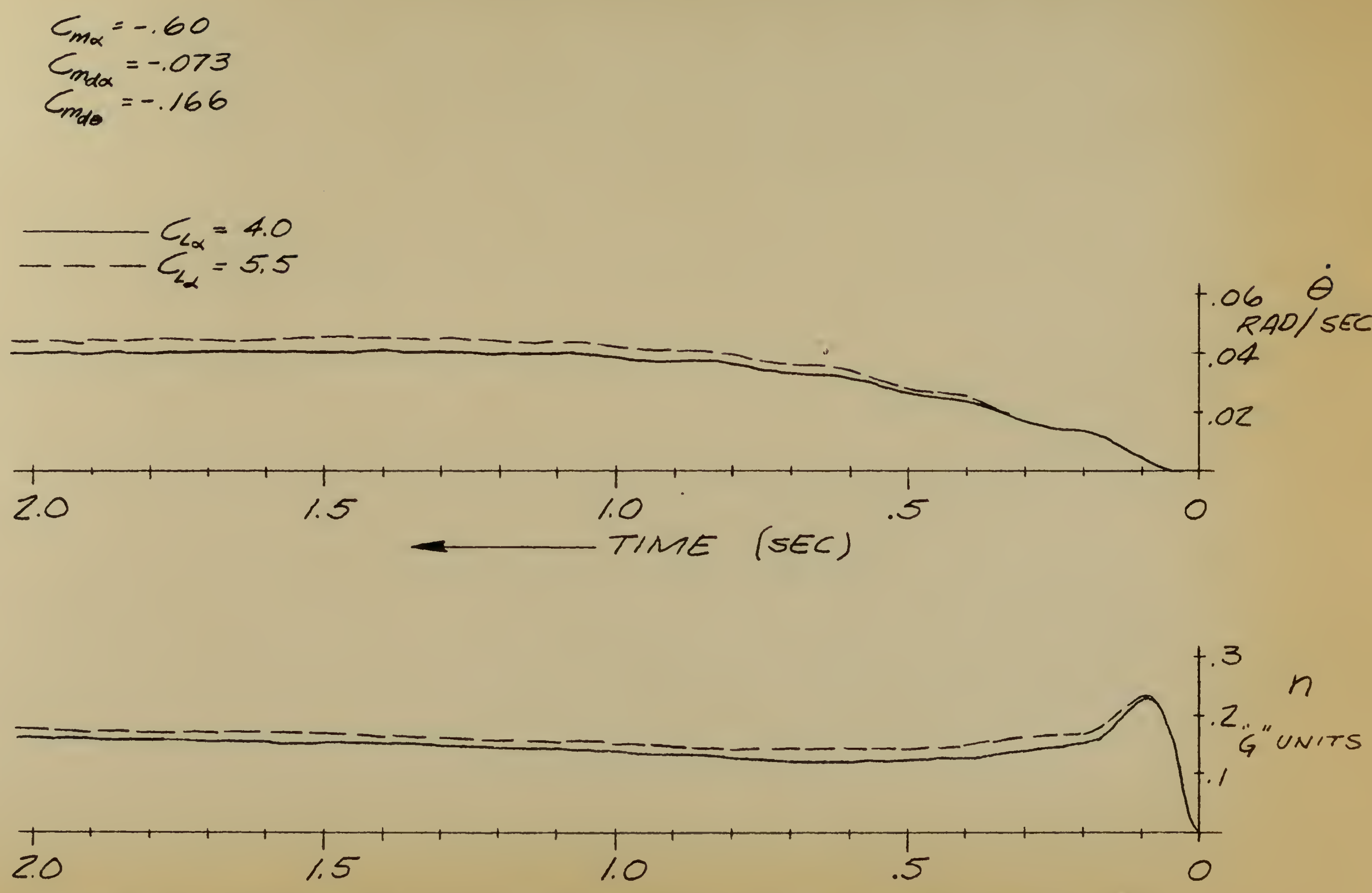


FIG. 20 ANALOG RESPONSES FOR VARYING $C_{L\alpha}$

BOMB DROP FORCING FUNCTION, $\Delta C_m = .0046$

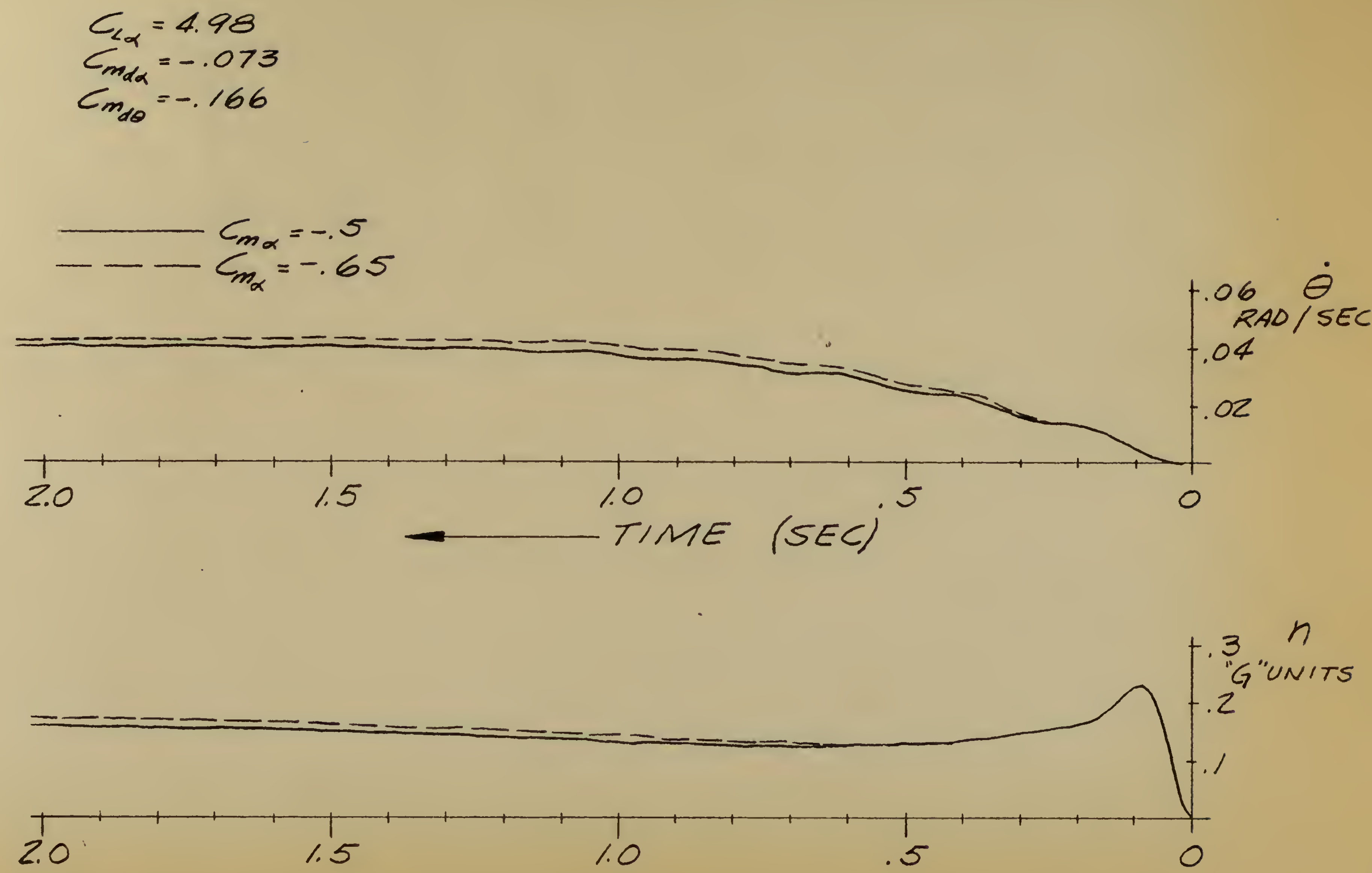


FIG. 21 ANALOG RESPONSES FOR VARYING C_{m_α}

BOMB DROP FORCING FUNCTION, $\Delta C_m = .0046$

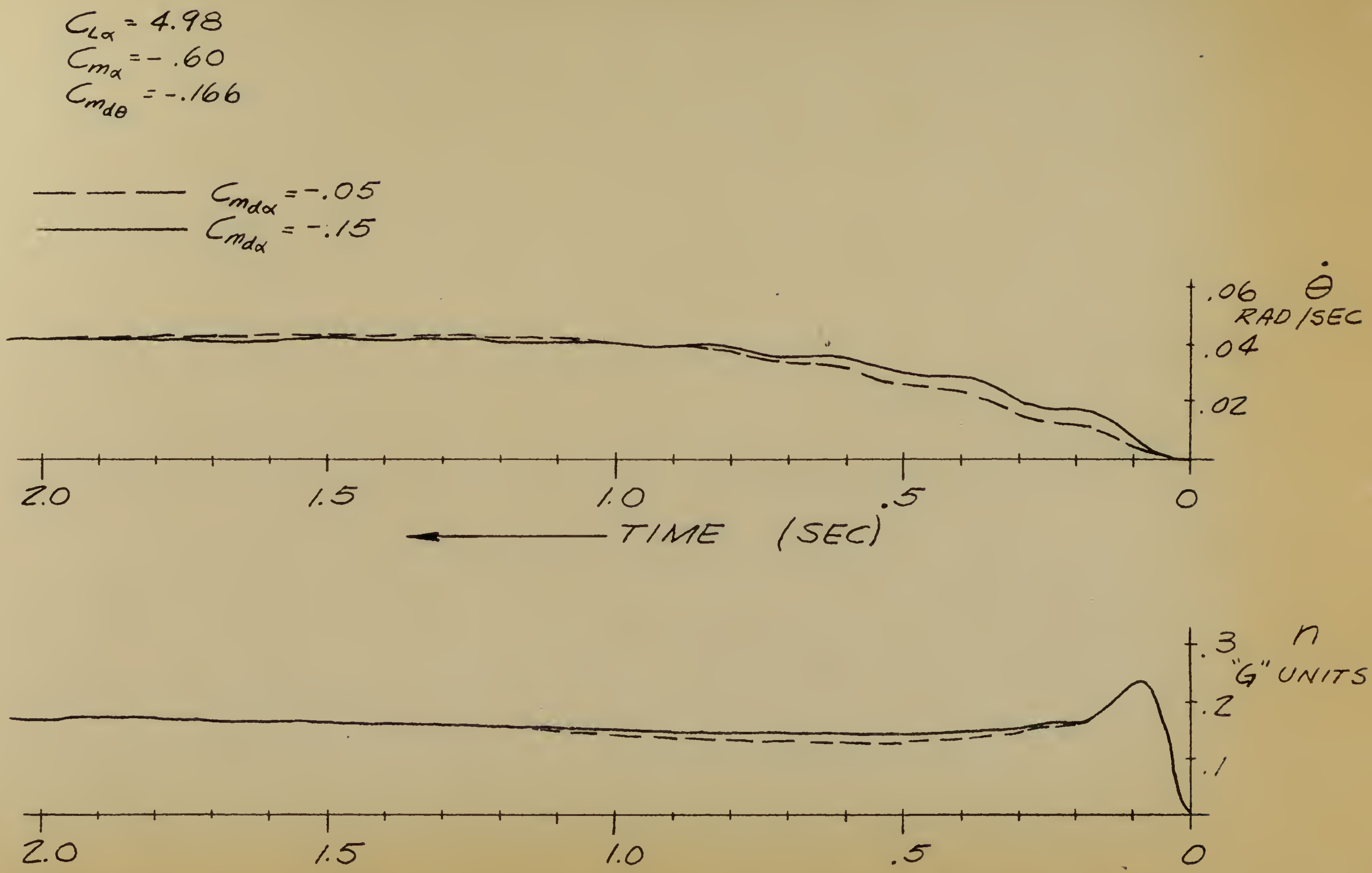


FIG. 22 ANALOG RESPONSES FOR VARYING $C_{m_{d\alpha}}$

BOMB DROP FORCING FUNCTION, $\Delta C_m = .0046$

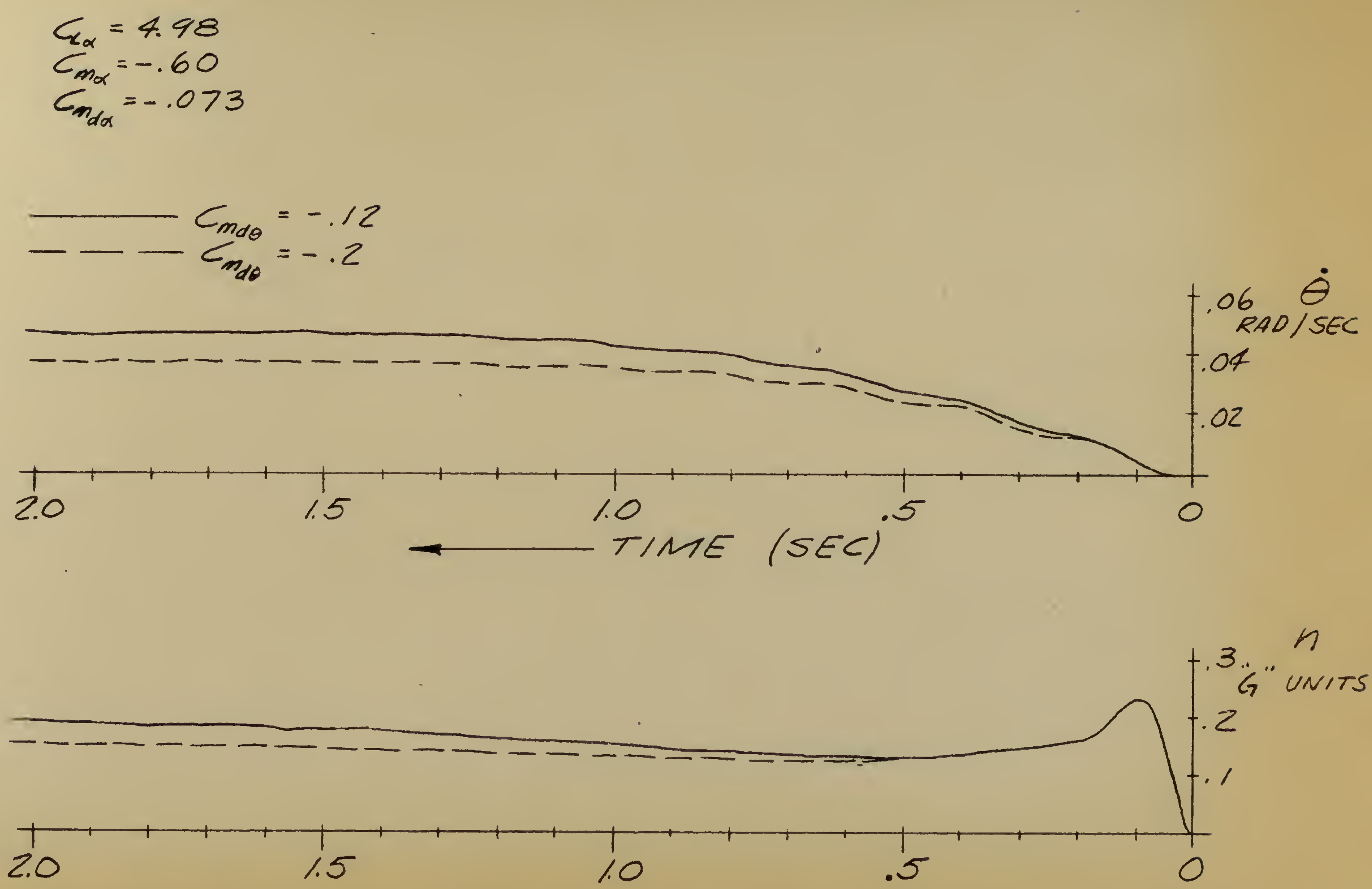


FIG. 23 ANALOG RESPONSES FOR VARYING $C_{m_{d\theta}}$

BOMB DROP FORCING FUNCTION, $\Delta C_m = .0046$

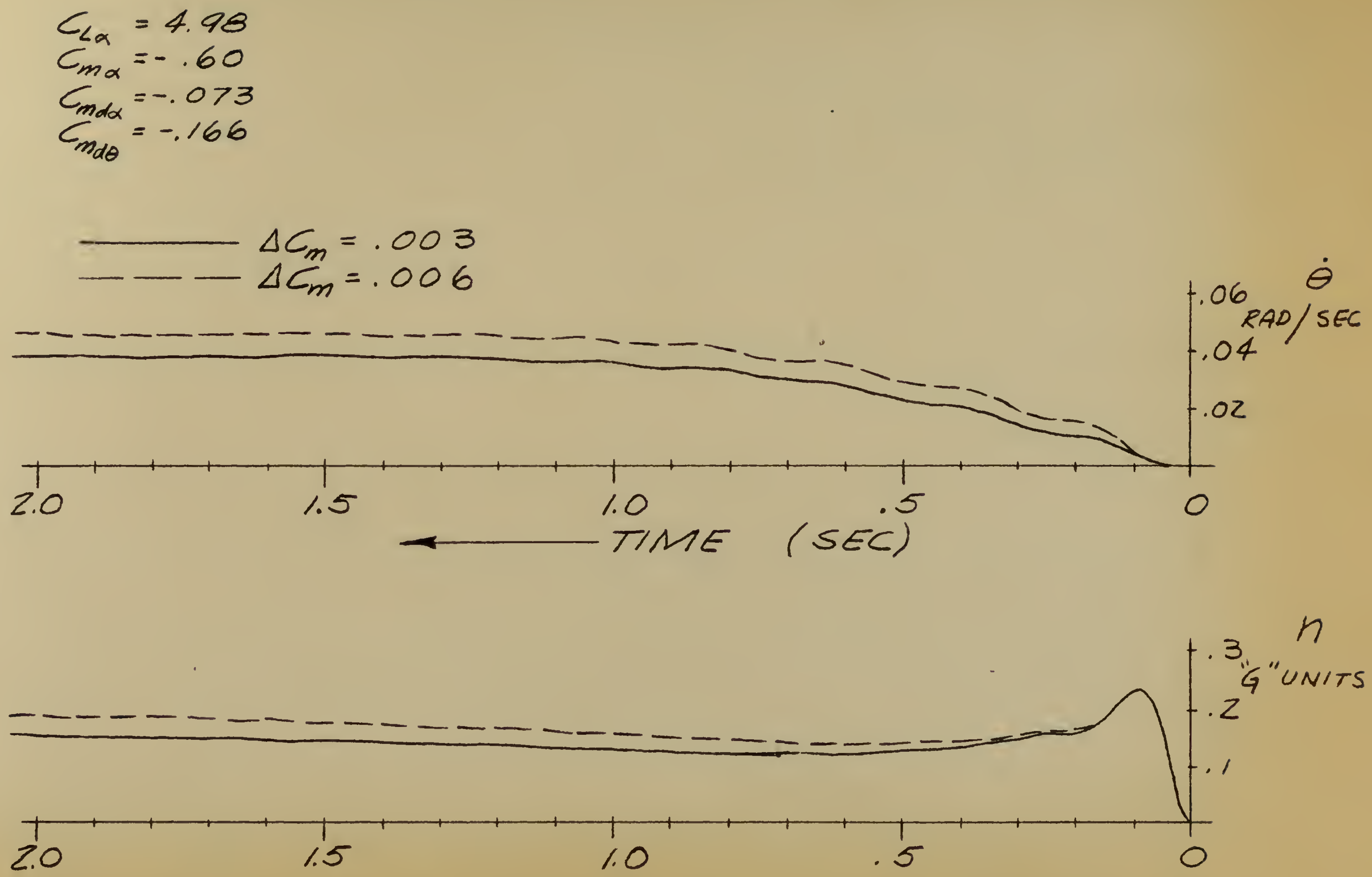


FIG. 24 ANALOG RESPONSES FOR VARYING ΔC_m

BOMB DROP FORCING FUNCTION

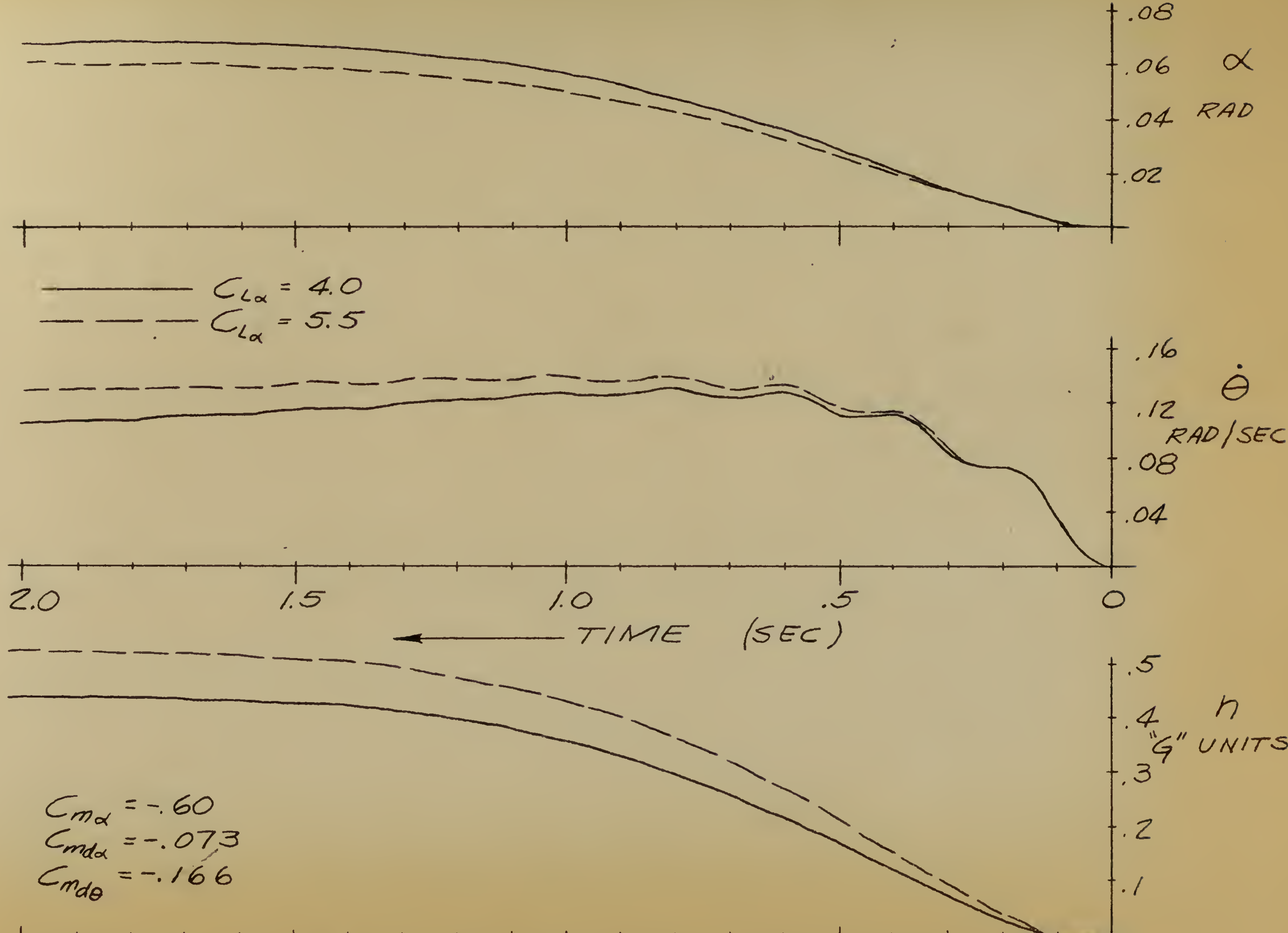


FIG. 25 ANALOG RESPONSES FOR VARYING $C_{L\alpha}$
ELEVATOR STEP FORCING FUNCTION

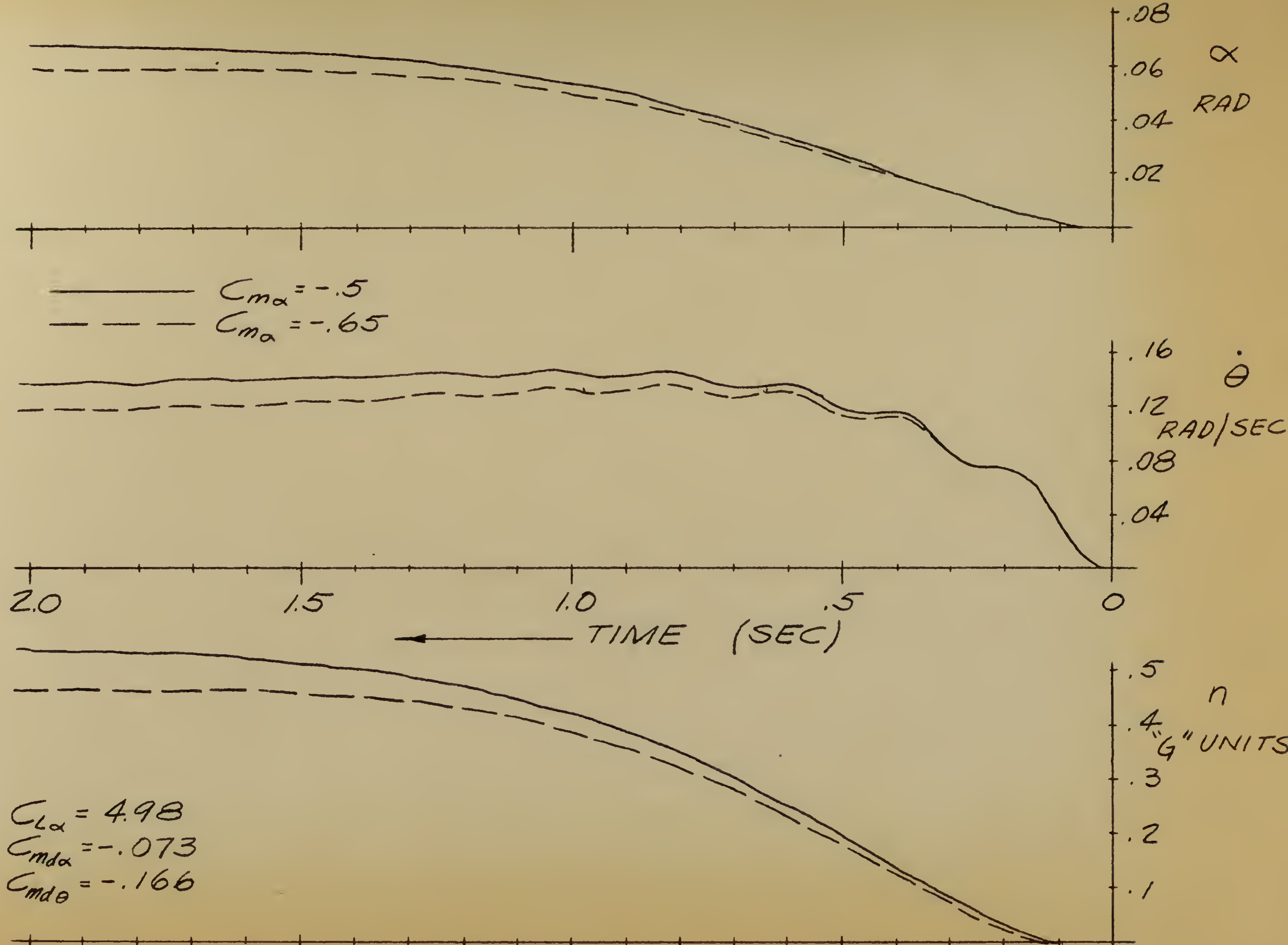


FIG. 26 ANALOG RESPONSES FOR VARYING $C_{m\alpha}$
ELEVATOR STEP FORCING FUNCTION

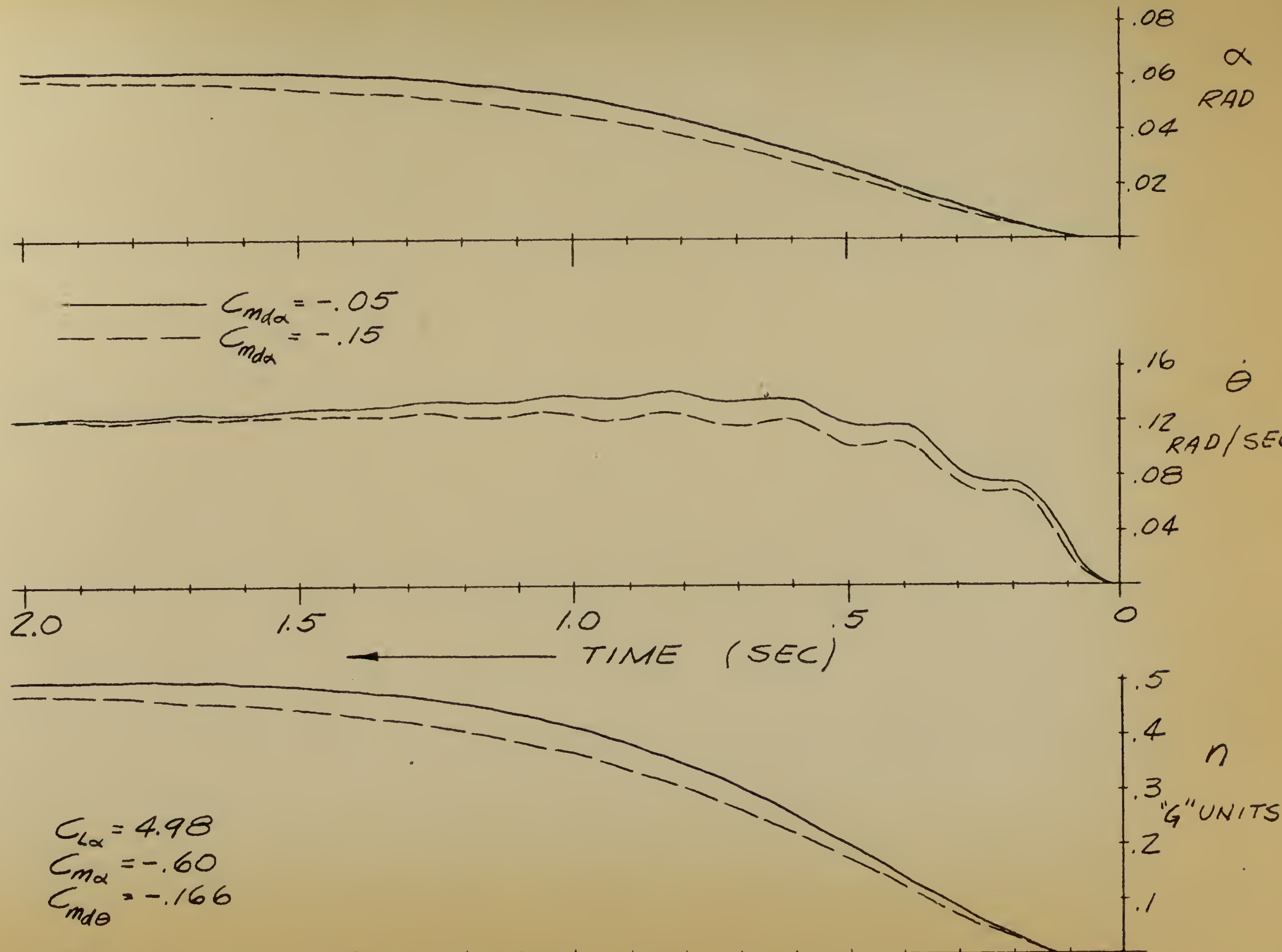


FIG. 27 ANALOG RESPONSES FOR VARYING $C_{m\dot{\alpha}}$
ELEVATOR STEP FORCING FUNCTION

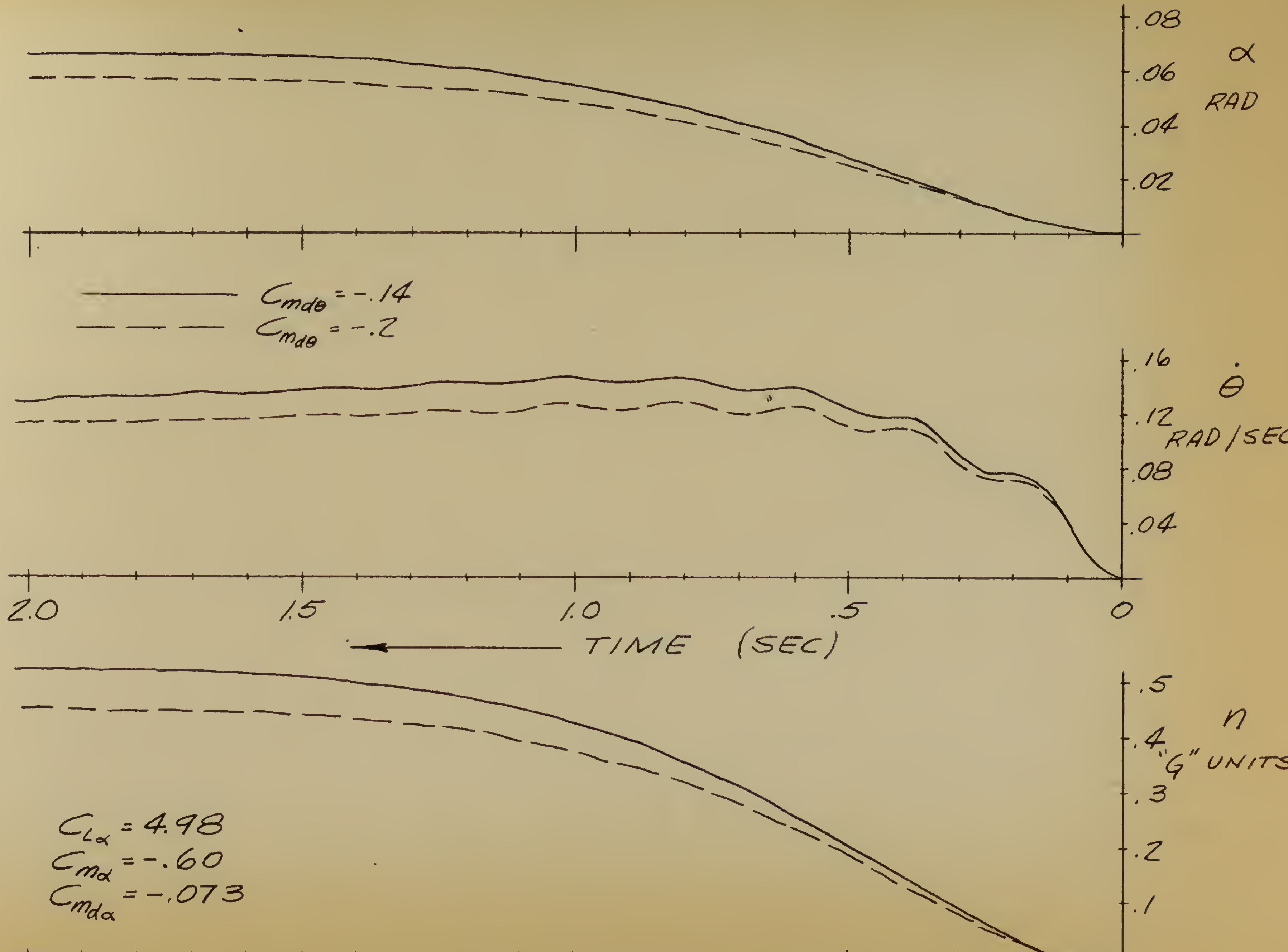


FIG. 28 ANALOG RESPONSES FOR VARYING $C_{m_{d\theta}}$
ELEVATOR STEP FORCING FUNCTION

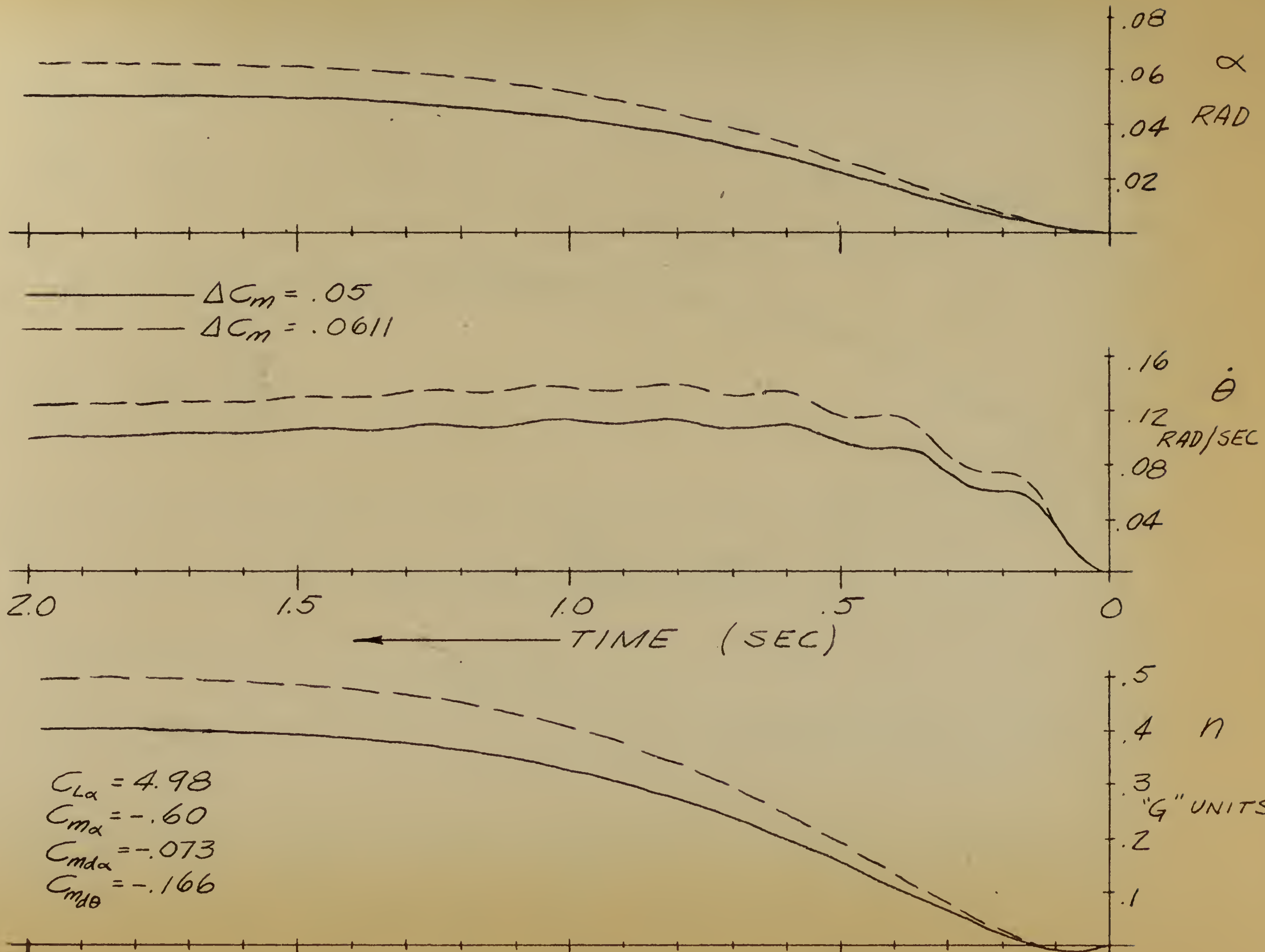


FIG. 29 ANALOG RESPONSES FOR VARYING ΔC_m
ELEVATOR STEP FORCING FUNCTION

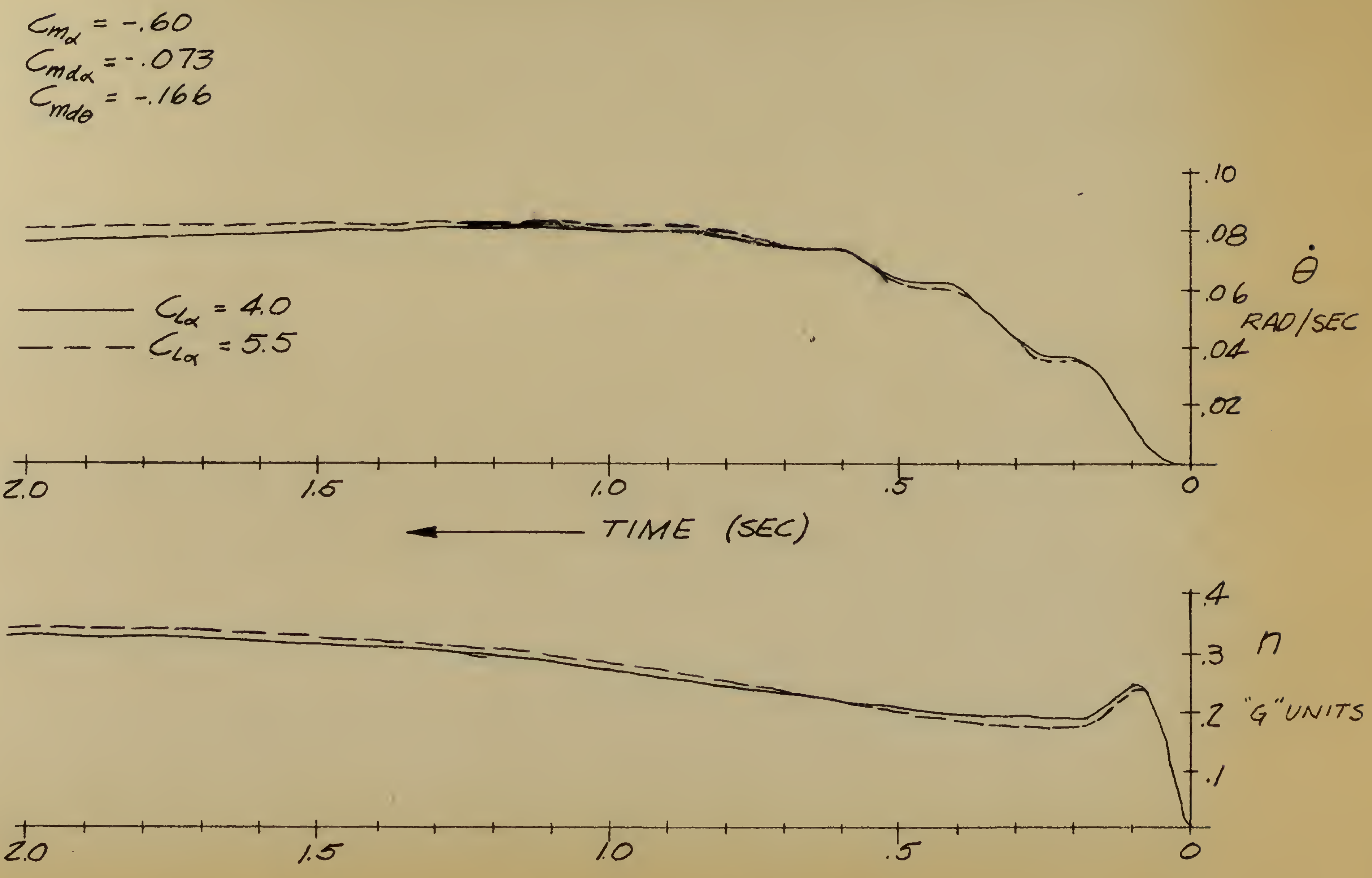


FIG. 30 ANALOG RESPONSES FOR VARYING C_{L_α}

BOMB DROP FORCING FUNCTION, $\Delta C_m = .0231$

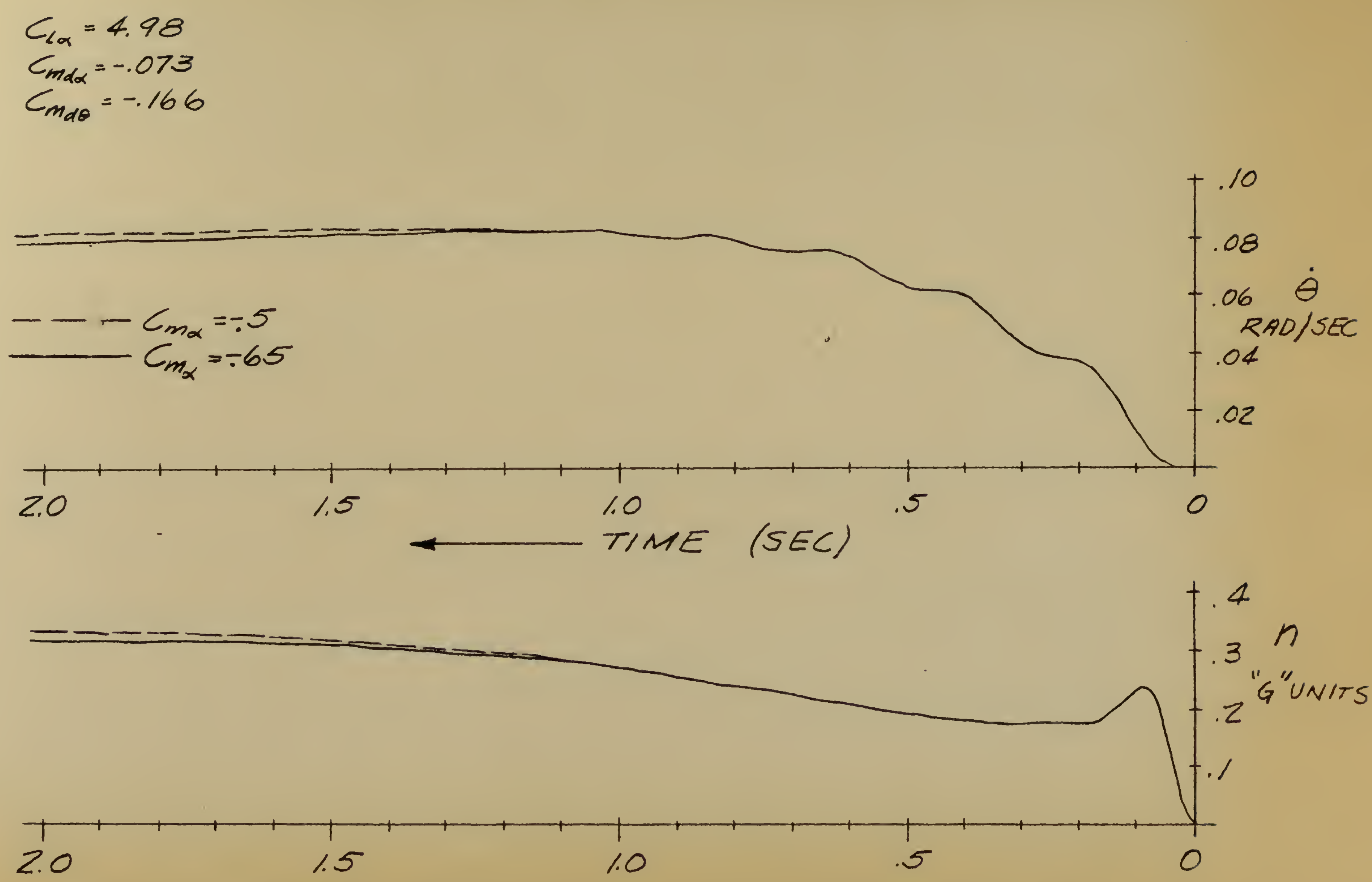


FIG. 31 ANALOG RESPONSES FOR VARYING C_{m_α}

BOMB DROP FORCING FUNCTION, $\Delta C_m = .0231$

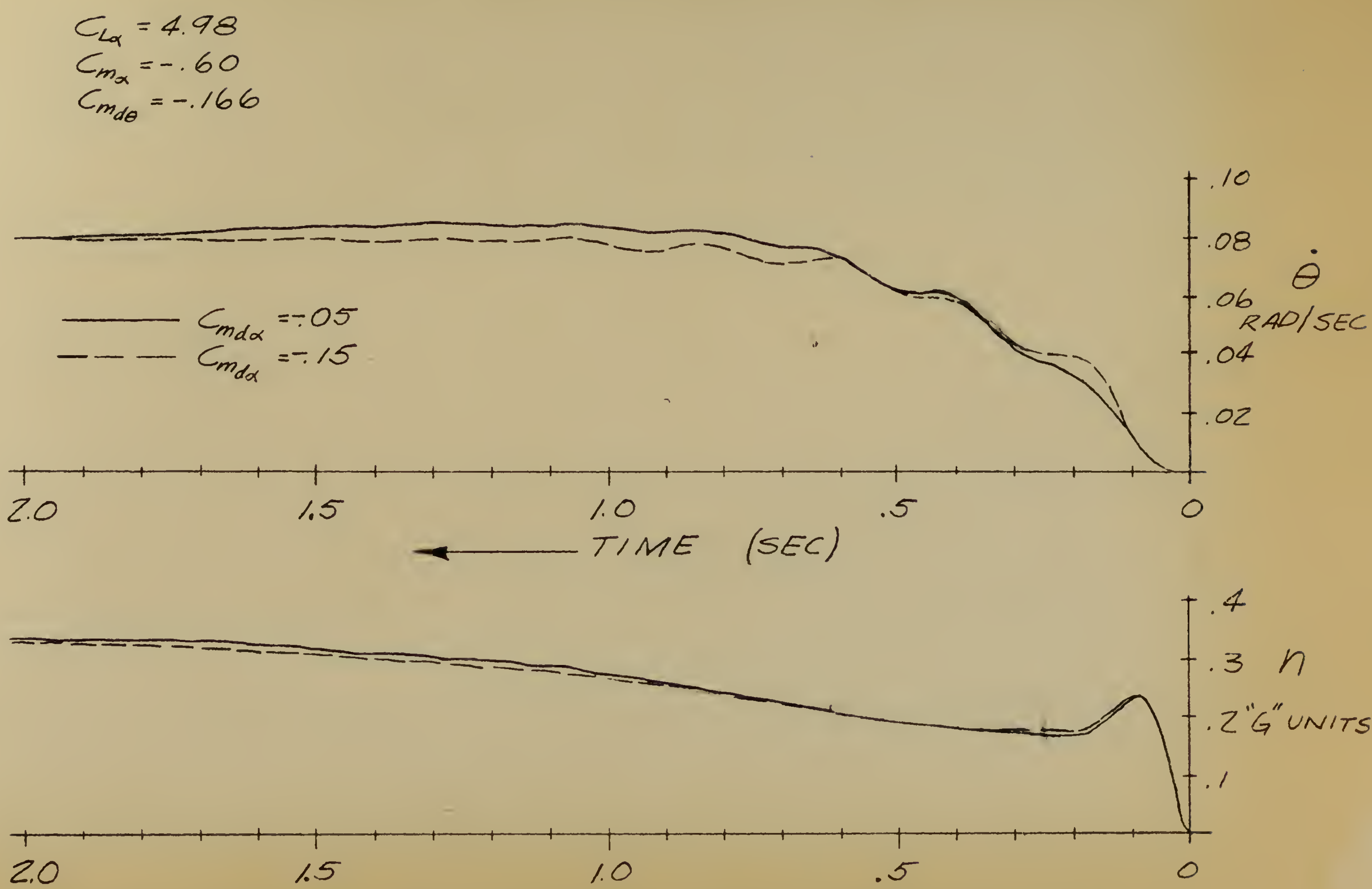


FIG. 32 ANALOG RESPONSES FOR VARYING $C_{m_{d\alpha}}$

BOMB DROP FORCING FUNCTION, $\Delta C_m = .0231$

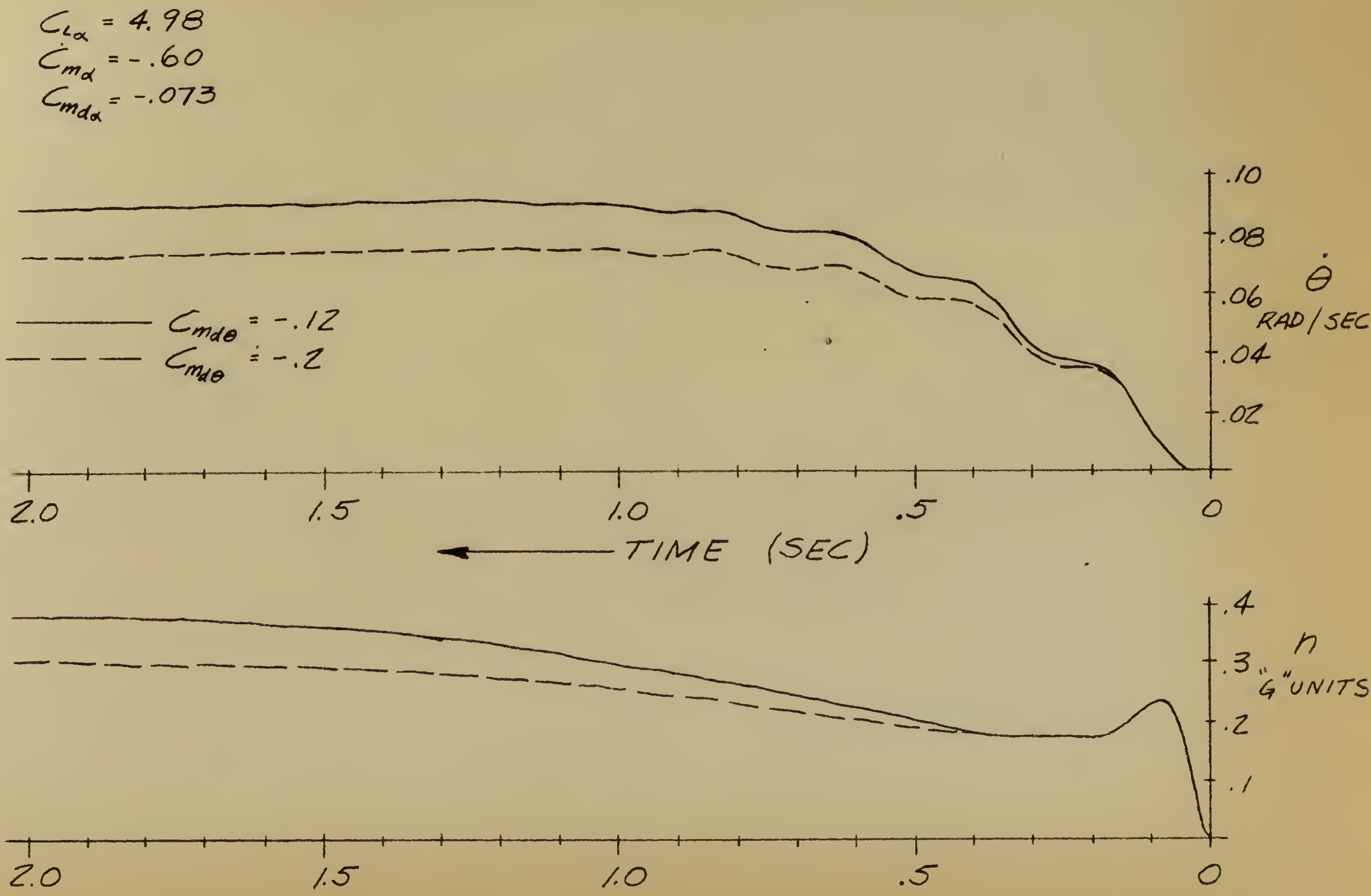


FIG. 33 ANALOG RESPONSES FOR VARYING $C_{m_{d\dot\theta}}$

BOMB DROP FORCING FUNCTION, $\Delta C_m = .0231$

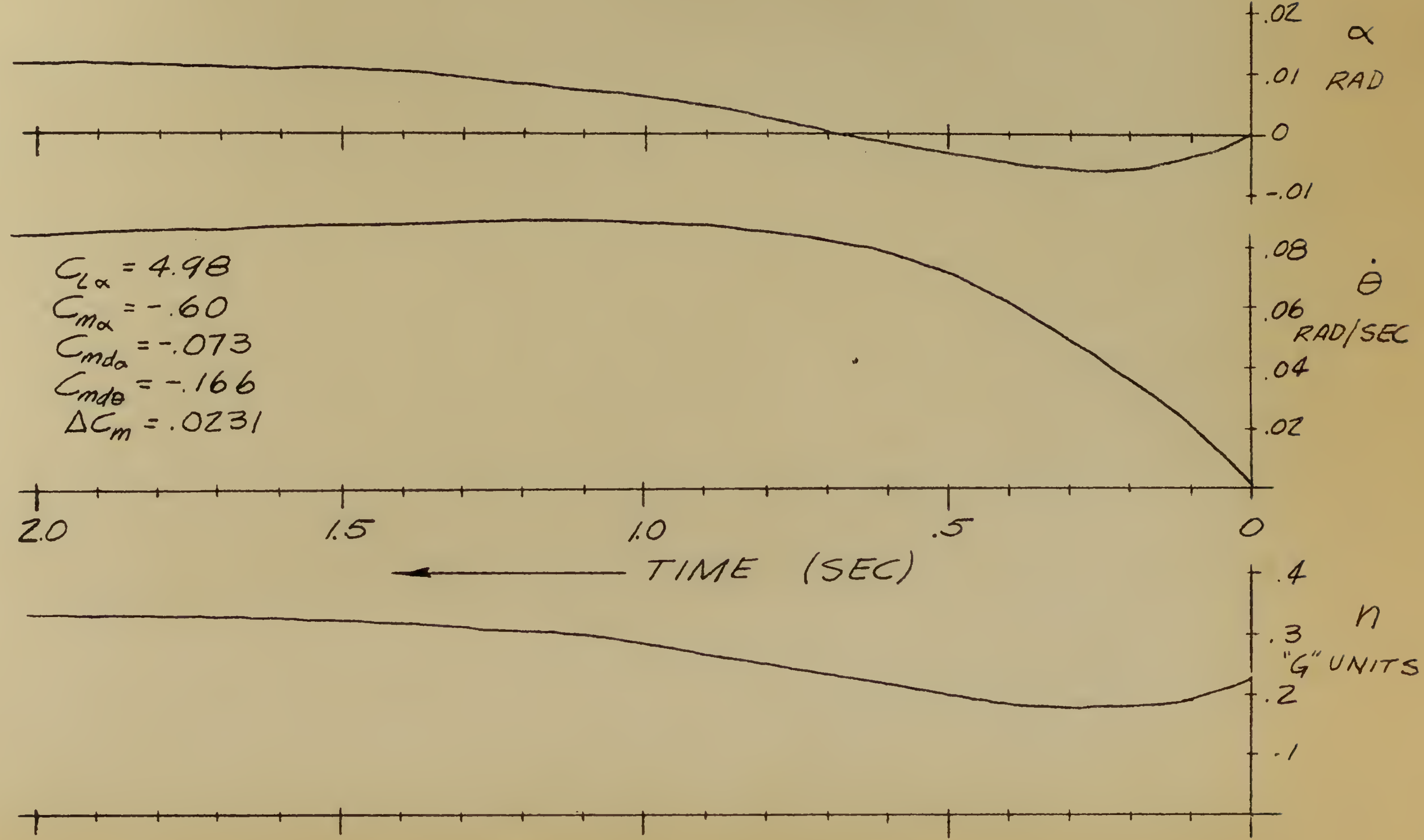


FIG. 34 ANALOG RESPONSE TO BOMB DROP EXCLUDING INSTRUMENT DYNAMICS